Reflection and transmission of twisted light at phase conjugating interfaces

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Abstract: We study the transmission and the reflection of light beams carrying orbital angular momentum through a dielectric multilayer structure containing phase-conjugating interfaces. We show analytically and demonstrate numerically that the phase conjugation at the interfaces results in a characteristic angular and radial pattern of the reflected beam, a fact that can be exploited for the detection and the characterization of phase conjugation in composite optical materials.

OCIS codes: (190.0190) Nonlinear optics; (190.5040) Phase conjugation.

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1. Introduction

In recent years light beams carrying orbital angular momentum (OAM) [1–7] have received an increased attention, particularly in the field of the optical manipulation and characterization of materials. Beams with OAM were realized as Laguerre Gaussian (LG) laser modes. These modes appear as annular rings with a zero on-axis intensity and are specified by the mode indices ℓ which are related to the angular degree of freedom around the propagation direction, and p that describes the number of nodes of the beam radial profile. This type of beams has been dubbed twisted light [8–10] because of the screw wave front, and also specified as an optical vortex with the topological charge or the winding number being equal to the integer value ℓ which may be either positive or negative depending on the sense of rotation of the screw wave front.

Recently, the behavior of LG beams at dielectric interfaces has been the subject of several studies showing that the beam-interface interactions are dependent on the incident angles of the LG beams. In the case of normal incidence, the azimuthal index of the reflected or the transmitted LG beam is increased and decreased by the cross-polarization coupling in the beam component of the incident beam [11–13]. In this contribution, we have study the propagation of twisted light in a multi layer dielectric medium containing interfaces that act as phase-conjugating mirrors (PCM) [14–17]. The problem for a single PCM has been addressed previously [14–17]. Here it is shown that for a structure containing several PCMs interferences lead to a characteristic angular and radial pattern of the reflected beam. This pattern can in turn serve as an indicator for the phase conjugation of a composite optical materials.

Theoretically, we evaluate the intensity of the reflected field after passing through phase conjugating interfaces (e.g. glasses doped with CdS_xSe_{1-x} [18] or other materials [19–20]). A detailed discussion of the reflection from a conventional mirror vs. a phase conjugating mirror

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was provided in Refs. [16,17,21,22] where it has been shown that the sign of the topological charge of a vortex beam is reversed and the OAM persists when it is reflected by a conventional mirror whereas the situation is different for a phase conjugating mirror due to its time reversal property, as a result of which, the incident and the reflected wavefront surfaces match perfectly. Therefore, the topological charge does not change sign and the optical OAM is reversed. Hence, the difference in angular momentum of $2\ell\hbar$ per photon is transferred to the phase-conjugating mirror. These remarkable image-transformation properties (even in the presence of a distorting optical element) are of interest for potential applications such as holography, mirror optics, optical fibers, optical trapping and many more [23]. These unique features of phase conjugating mirrors are confirmed by previous experimental studies [24–25].

2. Theoretical formulation

As sketched in Fig. 1, we consider a trilayer dielectric structure (the layers and related quantities are indexed by 0,1,2). All the layers are parallel and infinitely extended. The monochromatic LG beam with the frequency ω propagates in the medium 0 and impinges onto the medium 1. The interface between medium 0 and medium 1 as well as the interface between medium 1 and medium 2 are phase conjugating, d_1 is the thickness of the layer 1 (our treatment is also valid when the whole medium 1 is phase conjugating). We denote the incident, the reflected and the transmitted fields by $i,\ r$ and t respectively and $n_0,\ n_1$and n_2 are the refractive indices of the media 0, 1 and 2, respectively. We operate within the paraxial approximation, i.e. we assume that the transverse beam profile varies slowly along the direction of propagation. Beams with this property are collimated and have a well defined direction of propagation. We restrict the consideration to the case of a large Rayleigh range, corresponding to a well collimated beam with a relatively little divergence. The electric field E (at the beam waist, $z = 0$) of LG beam in cylindrical coordinates (with the z axis chosen to be along the incident beam propagation direction) is given by [26-27]:-

$$E = \frac{C_p^l}{w_0}\left(\frac{\sqrt{2}r}{w_0}\right)^{\ell} \exp\left(\frac{-r^2}{w_0^2}\right) L_p^{\ell}\left(\frac{2r^2}{w_0^2}\right) \exp i(k_0 nz - \omega t) \exp(i\ell\phi), \tag{1}$$

where, r and ϕ are the radial and azimuthal coordinates, ℓ can take any integer value either positive or negative and means physically the topological charge of the optical vortex. L_p^{ℓ} is the associated Laguerre polynomial, C_p^l is a normalization constant, w_0 is the half beam width, and $k_0 = \omega/c$ is the wave number in vacuum.

The well known time reversal property of phase conjugating mirror plays a key role in determining the behavior of the scattered beams electric fields. As detailed in Refs. [16–17]) the orbital angular momentum changes sign upon a wavefront reversal at pcm, i.e. ℓ changes sign (this goes with excitations in the pcm material such that the total angular momentum balance is guaranteed [16,17,30,31].) This ℓ property has to be imposed as an additional requirement on the beam when traversing the structure. To keep the notation simple we can incorporate this condition on ℓ by the ansatz

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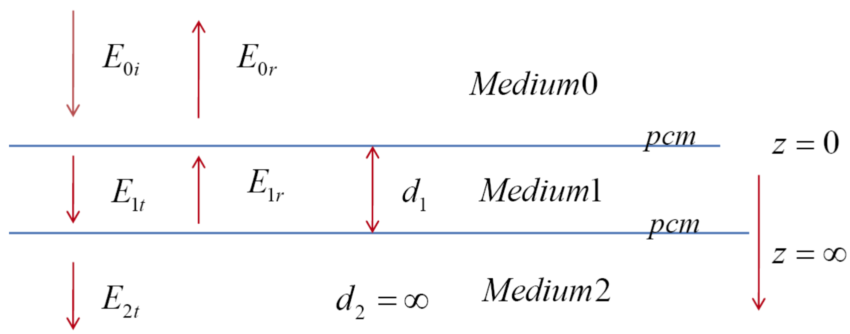

Fig. 1. Schematic representation of the propagation of LG beam in a multi layer dielectric structure. The interfaces with phase conjugation (pcm) are indicated.

$$E_{0i} = \bar{E}_{0i}e^{i\ell\phi} = \frac{C_p^l}{w_0}\left(\frac{\sqrt{2}r}{w_0}\right)^{\ell}\exp\left(\frac{-r^2}{w_0^2}\right)L_p^{\ell}\left(\frac{2r^2}{w_0^2}\right)e^{i(k_0n_0z+\ell\phi)}, (z\leq 0) \tag{2}$$

$$E_{0r} = \bar{E}_{0r}e^{-i\ell\phi} = r_0\frac{C_p^l}{w_0}\left(\frac{\sqrt{2}r}{w_0}\right)^{\ell}\exp\left(\frac{-r^2}{w_0^2}\right)L_p^{\ell}\left(\frac{2r^2}{w_0^2}\right)e^{-i(k_0n_0z+\ell\phi)}, (z\leq 0) \tag{3}$$

$$E_{1t} = \bar{E}_{1t}e^{i\ell\phi} = t_1\frac{C_p^l}{w_0}\left(\frac{\sqrt{2}r}{w_0}\right)^{\ell}\exp\left(\frac{-r^2}{w_0^2}\right)L_p^{\ell}\left(\frac{2r^2}{w_0^2}\right)e^{i(k_0n_1z+\ell\phi)}, (0\leq z\leq d_1) \tag{4}$$

$$E_{1r} = \bar{E}_{1r}e^{-i\ell\phi} = r_1\frac{C_p^l}{w_0}\left(\frac{\sqrt{2}r}{w_0}\right)^{\ell}\exp\left(\frac{-r^2}{w_0^2}\right)L_p^{\ell}\left(\frac{2r^2}{w_0^2}\right)e^{-i(k_0n_1z+\ell\phi)}, (0\leq z\leq d_1) \tag{5}$$

$$E_{2t} = \bar{E}_{2t}e^{i\ell\phi} = t_2\frac{C_p^l}{w_0}\left(\frac{\sqrt{2}r}{w_0}\right)^{\ell}\exp\left(\frac{-r^2}{w_0^2}\right)L_p^{\ell}\left(\frac{2r^2}{w_0^2}\right)e^{i[k_0n_2(z-d_1)+\ell\phi]}, (z\geq d_1). \tag{6}$$

Here, the temporal factor $\exp(-i\omega t)$ is omitted for the sake of simplicity. To evaluate the reflection and the transmission coefficients, we shall apply the condition of the continuity and smoothness of the field at the boundaries within the structure [28–29] (note the behavior of ℓ upon scattering is already accounted for by the ansatz (2-6))

$$[\bar{E}_{0i}+\bar{E}_{0r}]_{z=0} = [\bar{E}_{1t}+\bar{E}_{1r}]_{z=0}, \tag{7a}$$

$$[\bar{E}_{1t}+\bar{E}_{1r}]_{z=d_1} = [\bar{E}_{2t}]_{z=d_1}. \tag{7b}$$

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and

$$\left[\frac{\partial \bar{E}_{0i}}{\partial z}+\frac{\partial \bar{E}_{0r}}{\partial z}\right]_{z=0} = \left[\frac{\partial \bar{E}_{1t}}{\partial z}+\frac{\partial \bar{E}_{1r}}{\partial z}\right]_{z=0}, \tag{8a}$$

$$\left[\frac{\partial \bar{E}_{1t}}{\partial z}+\frac{\partial \bar{E}_{1r}}{\partial z}\right]_{z=d_1} = \left[\frac{\partial \bar{E}_{2t}}{\partial z}\right]_{z=d_1}. \tag{8b}$$

Eqs. (7) and (8) can be written as

$$1+r_0 = t_1+r_1, \tag{9a}$$

$$t_1 e^{i\alpha_1}+r_1 e^{-i\alpha_1} = t_2. \tag{9b}$$

and

$$n_0[1-r_0] = n_1[t_1-r_1], \tag{10a}$$

$$n_1[t_1 e^{i\alpha_1}-r_1 e^{-i\alpha_1}] = n_2 t_2. \tag{10b}$$

With the notation

$$\alpha_1 = k_0 n_1 d_1.$$

On solving Eqs. (9) and (10), we obtain the reflection coefficient r_0 related to the propagation in the medium 0, and the reflection and the transmission coefficients related to the propagation in the medium 1 and 2 denoted by r_1, t_1, r_2 and t_2, respectively.

Explicitly, the reflection and the transmission coefficients are

$$r_0 = \left(\frac{n_0A^- + n_1A^+}{n_0A^- - n_1A^+}\right), \tag{11}$$

$$r_1 = \frac{1+r_0}{1-e^{-2i\alpha_1}N}, \tag{12}$$

$$t_1 = -r_1 e^{-2i\alpha_1}N, \tag{13}$$

$$t_2 = \frac{n_1}{n_2}[t_1 e^{i\alpha_1} - r_1 e^{-i\alpha_1}]. \tag{14}$$

where

$$A^+ = 1+e^{-2i\alpha_1}N,$$

$$A^- = 1-e^{-2i\alpha_1}N,$$

$$N = \left(\frac{n_2+n_1}{n_2-n_1}\right).$$

After substituting for the reflection and the transmission coefficients in the Eqs.(2)-(6), we can obtain the electromagnetic fields that describe the propagation of the LG beam through the system depicted in Fig. 1.

3. Results

Experimentally we imagine a situation where the reflected beam perpendicular to the structure (i.e. along the $-z$ axis) is detected. This case has been studied recently experimentally and theoretically for a single pcm in [14–17]. The results of these studied are recovered from the above formula as a special case when only one pcm is present. In a multi layer structure interference effects are to be expected. Of a particular interest for us are those effects which are related to

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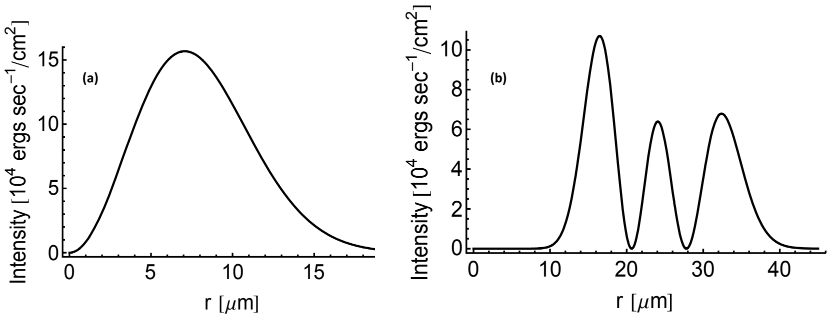

Fig. 2. For the structure depicted in Fig. 1 we show the calculated total radial (r) intensity (in CGS system) of the LG laser beam in the medium 0 for $\ell = 1$, $p = 0$ (a), and for $\ell = 10$, $p = 2$ (b). The material parameters and laser properties are chosen as: $\phi = 30^\circ$, $n_0 = 1$(air), $n_1 = 1.77$ (Al_2O_3), $n_2 = 1.457$ (SiO_2), $d_1 = 20\ \mu$m, w_0=1 μm, $\lambda = 632.9$ nm.

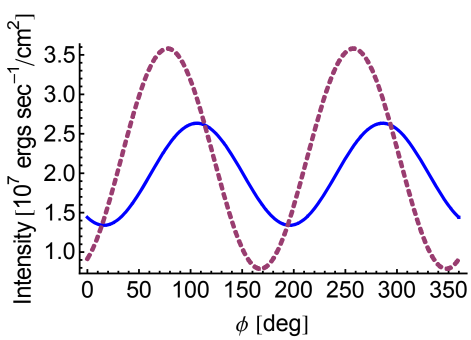

Fig. 3. The same as in Fig. 2 for $\ell = 1$, $p = 0$ but here we show the angular (ϕ) distribution of the LG beam intensity (in CGS system) for a different thickness d_1 of the medium 1. The blue solid curve is for $d_1 = 11\pi\lambda/2$ and the dashed curve is for $d_1 = 4\pi\lambda$. The radial distance r is fixed to be $w_0/2$.

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ℓ and the phase conjugating properties, say of the interface to medium 2. The idea is to infer from the detected reflected beam on the properties of the inhomogeneities (medium 1) in a bulk optical material. To this end we show in Fig. 2 the radial distribution of the total intensity of the reflected field (Eq. (3)) from the structure in Fig. 1 for a LG laser with $\lambda = 632.9$ nm. We consider a situation where the medium 0 is air ($n_0 = 1$), medium 1 is Al_2O_3 ($n_1 = 1.77$), and medium 3 is SiO_2 ($n_2 = 1.457$). The thickness of Al_2O_3 is $d_1 = 20\ \mu$m. A very thin pcm is deposited on Al_2O_3 and another thin pcm is deposited on SiO_2. Figs.2 show that the reflected beam maintains the initial beam shape upon traversing the whole structure for different values of ℓ & p. As discussed in Refs.[16,17] the angular distribution of the LG beam leads a special interference pattern upon reflection from one pcm. Qualitatively we can thus expect a characteristic change of this pattern when the LG beam reflected from the interface to medium 2 contributes to the total reflected intensity. Obviously, this change depends on the thickness d_1 and varies on the scale of the wave length of the incoming LG beam. This behavior is confirmed by Fig. 3 from which we can conclude that the reflected LG carries depth information on phase conjugating inhomogeneity.

4. Conclusion

We studied the propagation of a light beam carrying orbital angular momentum (OAM) in a dielectric multi layer structure with phase conjugating properties. Analytical expressions for the reflection and the transmission of the fields at individual layers are provided. We demonstrated that the scattering of OAM beams from phase conjugating refractive inhomogeneities leads to characteristic interferences that depend on the depth profile. This can be tested by, e.g. varying the light wave length.

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