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Indirect exchange interaction between two local spins embedded in an Aharonov–Bohm Ring

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Abstract

We examine the Ruderman–Kittel–Kasuya–Yosida (RKKY) interaction between two local spins embedded in an Aharonov–Bohm (AB) ring. The flux-dependent RKKY interaction dominates the transport properties for both ferromagnetic and antiferromagnetic coupling. For ferromagnetic coupling, the amplitude of AB oscillations is enhanced by Kondo correlations and for antiferromagnetic coupling the phase of AB oscillations is shifted by π.

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1. Introduction

Two local spins embedded in a metal couple magnetically even they are spatially separated. Such indirect exchange interaction, the Ruderman–Kittel–Kasuya–Yosida (RKKY) interaction, has been known from the 1950s [1] and is one of basic mechanisms for spintronics in magnetic nanostructures [2,3]. For a semiconducting nanostructure, an experiment on two quantum dots (QDs) coupled indirectly thorough an open QD shows a signature of the possible RKKY interaction [4]. It stimulated theoretical discussions [5–7]. Earlier, we have analyzed theoretically the RKKY interaction for an Aharonov–Bohm (AB) ring embedded with a QD in each arm [8] (inset of Fig. 1). The idea is based on the long Fermi wave length $\lambda_{\rm F}$ of a two-dimensional (2D) electron gas at GaAs/AlGaAs interface (typically of the order 100 nm) and the slow decay of the RKKY interaction $\sim\lambda_{\rm F}/(2\pi l)$ for 1D lead, where l is the distance between two QDs.

2. Flux-dependent RKKY interaction

QDs with odd numbers of electrons behave local spin $\frac{1}{2}$ and the effective Hamiltonian of our system consists of two single-channel 1D leads H_0 and a tunneling part $H_{\rm T}$: $H_0=\sum_{kr\sigma}\varepsilon_k a^\dagger_{kr\sigma}a_{kr\sigma}$, where $a_{kL(R)\sigma}$ is an annihilation operator of an electron with quantum number k and spin σ in the left (right) lead. $H_{\rm T}=\sum_{r,r'=L,R}\sum_{n=1,2}\sum_{\sigma,\sigma'=\uparrow,\downarrow}(J/2)a^\dagger_{rn\sigma}\tau_{\sigma,\sigma'}S_n a_{r'n\sigma'}$, where S_n describes the spin state of nth QD, τ is the Pauli matrix, and $J>0$ is an antiferromagnetic (AF) coupling constant between a QD spin and a spin in a lead. The operator $a_{rn\sigma}$ annihilates an electron in lead r at the boundary of nth QD. When the magnetic field is applied, an AB phase factor, ${\rm e}^{\pm {\rm i}\phi/2}$ ($\phi=2\pi\Phi/\Phi_0$, where $\Phi_0=hc/e$ is the flux quantum), should be counted in $H_{\rm T}$, when an electron tunnels through a QD in the clockwise/

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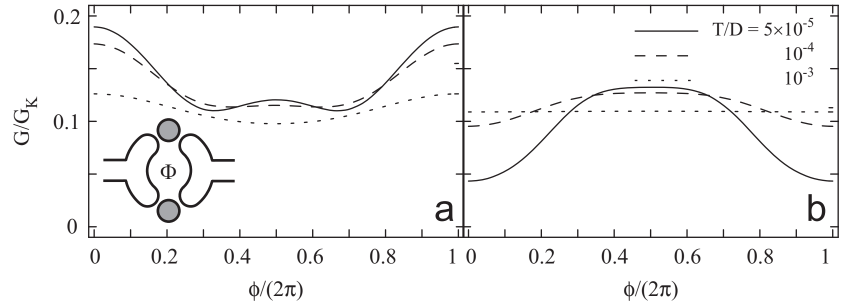

Fig. 1. Flux-dependent conductance for ferromagnetic coupling (a) and (b) for antiferromagnetic coupling (the Kondo temperature is $T_{\rm K}/D \approx 10^{-6}$). Inset: Aharonov–Bohm ring embedded with one QD in each arm.

anticlockwise direction. The AB flux is written with the vector potential as, $\Phi = \oint A\mathrm{d}l$ where the line integral is performed along the ring in the clockwise direction.

The RKKY interaction is obtained by the second order perturbation theory in terms of J [1]: $H_{\rm RKKY} = (J_{\rm RKKY}(\phi)/2)\, S_1\, S_2$,

$$J_{\rm RKKY}(\phi) = (J^2/2)\chi(2 + 2\cos\phi),$$

$$4J^2\chi = {\rm Re}\int_{-D}^{0} \mathrm{d}\varepsilon\, \mathrm{d}\varepsilon' \sum_{p=+,-} \frac{\gamma_p(\varepsilon)\gamma_p(\varepsilon') - \gamma_p(\varepsilon)\gamma_{\bar{p}}(\varepsilon')}{\varepsilon - \varepsilon' + \mathrm{i}0}, \qquad (1)$$

where $\bar{p} = \pm$ for $p = \mp$. A phase-dependent $2\cos\phi$ is related to two configurations of particle–hole excitations, which enclose the flux and pick up a phase factor $\mathrm{e}^{\mathrm{i}\phi}$ or $\mathrm{e}^{-\mathrm{i}\phi}$. A spectral function of even/odd parity "electron propagator" is $\gamma_\pm(\varepsilon) \simeq J\rho[1 \pm \cos\{k_{\rm F}l\,(1 + \varepsilon/D)\}]$ within the quasiclassical approximation, where $D = \hbar v_{\rm F}k_{\rm F}$ is the cut-off energy. The argument of cosine function is the accumulated phase during the electron propagation between two QDs. The expression Eq. (1), obtained for $T = 0$, is approximately valid below the Thouless energy $E_{\rm Th} \equiv \hbar v_{\rm F}/l$, where $J_{\rm RKKY}(\phi)$ oscillates and decays as a function of $k_{\rm F}l$, $J_{\rm RKKY}(\phi) \simeq -\pi(J\rho)^2 D\cos(2k_{\rm F}l)(1+\cos\phi)/(2k_{\rm F}l)$.

3. Kondo correlations

Due to the RKKY interaction, the two dot spins are entangled and form a singlet state $|0,0\rangle$ for AF coupling or a triplet state $|1,m\rangle$ $(m = 0, \pm 1)$ for ferromagnetic (F) coupling. Probabilities of the singlet state P_0 and each of particular triplet states P_1 depend on the flux $P_0 = 1/\{1 + 3\exp(-J_{\rm RKKY}(\phi)/T)\}$ and $P_1 = (1 - P_0)/3$. The effect of the RKKY interaction would be pronounced for temperatures $T < |J_{\rm RKKY}(0)|$. Typically at low temperature, Kondo correlations are also relevant, since they start to grow much above the Kondo temperature $T_{\rm K} \approx D\exp\{-1/(2J\rho)\}$. Therefore, we calculate the conductance perturbatively in terms of J up to the third order using the real-time diagrammatic technique [8].

Here, we will discuss three special cases (i) the uncorrelated local spins ($|J_{\rm RKKY}| \ll T$) (ii) the F coupling ($-J_{\rm RKKY} \gg T$), and (iii) the AF coupling ($J_{\rm RKKY} \gg T$) cases. The case (i) is realized around a half-integer flux $\phi = \pi$ or for high temperature. Then the local-spin state is distributed with equal probability $P_1, P_0 \approx \frac{1}{4}$, and the conductance is $G/G_{\rm K} \simeq (\pi J\rho)^2[\{1 + \cos\phi\cos^2(k_{\rm F}l)\} + 3(1 + 4J\rho\ln\{2\mathrm{e}^{\gamma}D/(\pi T)\})]$. The first term is attributed to the cotunneling process preserving spin and therefore showing AB oscillations. The background, the second term, comes from spin–flip processes enhanced by Kondo correlations.

The case (ii) is important around integer values of $l/\lambda_{\rm F}$ at low temperatures, where two local spins form a triplet state $P_1 \approx \frac{1}{3}$ and $P_0 \approx 0$. Then the spin-1 Kondo physics, $G/G_{\rm K} \simeq (\pi J\rho)^2[8J\rho\,\cos^2(k_{\rm F}l)\,\cos^2(\phi/2)\,\ln\{2E_{\rm Th}/(\pi T)\} + (3 + 4J\rho\,\ln\{2\mathrm{e}^{\gamma}D/(\pi T)\})\{1 + \cos\,\phi\,\cos^2(k_{\rm F}l)\}]$ is dominant (γ is the Euler constant). As opposed to the case (i), Kondo correlations enhance the oscillatory component as shown in the second term. It is because, two spins are no longer independent phase-breaking scatterers they "observe" each other and Kondo correlations just enhance coupling J. The first term is related to the fact that the spin-1 moment stretches over l: it shows the logarithmic enhancement with the Thouless energy $E_{\rm Th}$.

In the case (iii), two local moments form a singlet state $P_1 \approx 0$ and $P_0 \approx 1$. As the singlet state is decoupled from lead electrons, i.e. electrons flowing through QDs cannot excite the system from the singlet state to a triplet state, only the spin preserving cotunneling process contributes to the conductance: $G/G_{\rm K} \simeq (\pi J\rho)^2\{1 + \cos\,\phi\,\cos^2(k_{\rm F}l)\}$.

Fig. 1 shows the conductance for various temperatures as a function of the flux ϕ. The panels (a) and (b) are for F ($k_{\rm F}l/\pi$ is an integer) and for AF ($k_{\rm F}l/\pi$ is a half-integer) coupling cases. In the vicinity of zero or one flux, the maximum ferromagnetic RKKY interaction is induced. For F coupling case, a triplet state is formed at low temperature and the conductance is enhanced (ii). For AF coupling case, at low temperature, the singlet is formed and the conductance is suppressed (iii). At half flux, the RKKY interaction is switched off, but we can observe the maximum in the conductance for both panels. According to discussion in (i), this maximum corresponds to incoherent transport thought the two independent spin-$\frac{1}{2}$ local moments enhanced by Kondo correlations. By combining such behaviors, for AF coupling case, the phase of AB oscillations is shifted by

π, and for F coupling case an additional maximum appears at half-integer values of the flux.

4. Summary

We have theoretically investigated the interplay between RKKY interaction and Kondo correlations for two QDs embedded in the AB ring. The flux can rule the RKKY interaction by controlling the interference: around integer (half-integer) values of flux, the RKKY interaction is maximum (switched off). As a result, for AF coupling case, the phase of AB oscillations is shifted by π, and for F coupling case, an additional maximum appears at half-integer values of flux due to Kondo correlations.

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References

[1] C. Kittel, Solid State Physics, vol. 22, Academic Press, New York, 1968.
[2] S. Maekawa and T. Shinjo (Eds.), Spin Dependent Transport in Magnetic Nanostructures, Taylor & Francis, London, 2002.
[3] P. Bruno, Phys. Rev. B 52 (1995) 411.
[4] N.J. Craig, J.M. Taylor, E.A. Lester, C.M. Marcus, M.P. Hanson, A.C. Gossard, Science 304 (2004) 565.
[5] M.G. Vavilov, L.I. Glazman, Phys. Rev. Lett. 94 (2005) 086805.
[6] P. Simon, R. López, Y. Oreg, Phys. Rev. Lett. 94 (2005) 086602.
[7] H. Tamura, K. Shiraishi, H. Takayanagi, Phys. Rev. B 72 (2005) 121308 (R).
[8] Y. Utsumi, J. Martinek, P. Bruno, H. Imamura, Phys. Rev. B 69 (2004) 155320.