TRANSPORT PROPERTIES OF DOMAIN WALLS IN FERROMAGNETS

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The influence of domain walls on transport properties of ferromagnetic materials is analyzed theoretically and the results are compared with recent experiments. In the case of diffusive transport through a thick domain wall, the semiclassical approximation is applied and a local spin transformation is performed, which replaces the system with a domain wall by the corresponding system without domain wall but with an additional gauge field. Due to a redistribution of single-particle electron states at the wall, one obtains either negative or positive contributions to resistivity. On the other hand, suppression of the weak localization corrections to conductivity by the gauge field created by the wall leads to an increase in the low-temperature conductivity in the presence of a domain wall. In the case of narrow domain walls the semiclassical approximation is not valid. Instead of this one can use an approach based on scattering matrix. In this particular case, the domain wall induces a large positive contribution to the resistivity. The corresponding magnetoresistance in nanostructures with sharp domain walls can be large, in accordance with recent experiments.

1. Introduction

It is well know for long time that magnetic domain walls (DWs) in a ferromagnetic metal influence its electronic transport properties by producing an additional contribution to electrical resistivity. Since DWs give rise to electron scattering,[1,2] one could expect that this contribution is positive. This expectation was also supported by early experiments. It was only very recently when a single DW contribution to electrical resistivity could be extracted in a controllable way from the overall resistance.[3,4,5,6] Surprisingly, it turned out that the resistance of a system with DWs in some cases was smaller than in the absence of DWs,[3,4] whereas in other cases it was larger.[7,8,9] This intriguing observation led to considerable theoretical interest in electronic transport through DWs.[10,11,12,13,14] The interest is additionally stimulated by possible applications of the associated magnetoresistance in magnetoelectronics devices. This is because creation and destruction of DWs can be controlled by a weak magnetic field. The corresponding magnetoresistance can be then either positive or negative.

Recent experiments on magnetic point contacts showed that constrained DWs formed at the very contact between ferromagnetic wires produce an unexpectedly large contribution to electrical resistivity, and consequently lead to large negative magnetoresistance.[15] The characteristic feature of DWs in point contact geometry is their very small width (a few angstroms),[16,17] which is much smaller than the DWs width in bulk materials, thin films, or in wires.

In the following we will describe theoretically basic features of the electronic transport through DWs, and will present explanation of the above described experimental observations. Two limits will be analyzed in detail – the limit of thick DW, when electronic transport through the wall is diffusive, and the limit of narrow DW, when the transport is ballistic. In the former case the theoretical treatment is based on a semiclassical approach, which is valid for $k_{F\uparrow(\downarrow)}D \gg 1$, where $k_{F\uparrow}$ and $k_{F\downarrow}$ are the Fermi wavevectors corresponding to the two spin channels, and D is a characteristic length of the magnetization variation (DW width).[18] In such a case DW can lead to redistribution of single-electron quasiparticles, and this can lead either to positive or to negative contribution to resistivity. Another mechanism which leads to negative contribution is based on the suppression of weak localization (WL) corrections to conductivity by DWs.[10] At sufficiently low temperatures quantum interference effects in a magnetically uniform system (without DWs) lead to an increase in the resistivity due to enhanced back scattering.[19,20] Creation of DWs destroys the interference effects and therefore diminishes resistivity of the system.

When, however, the DW width D is of atomic size, like in some nanoconstrictions,[16] the condition of semiclassical behavior is not fulfilled. In that case, one has to use a different approach, like for instance the one based on the scattering matrix and Landauer formalism.

2. Diffusive transport through a thick domain wall

2.1. *Model*

Assume a simplified model of a ferromagnetic metal, in which conduction electrons with a parabolic energy spectrum interact with a nonuniform magnetization that smoothly varies across a certain DW. Assume also that the electrons are scattered by defects with the corresponding scattering potential being independent of the spin orientation (in a general case this potential can be spin dependent). When the domain wall is sufficiently thick, $D >> l$, where l is the electron mean free path, electronic transport across the wall is diffusive.

The single-particle Hamiltonian describing conduction electrons locally exchange-coupled to the magnetization $\mathbf{M}(\mathbf{r})$ takes the form

$$H_0 = -\frac{1}{2m}\frac{\partial^2}{\partial \mathbf{r}^2} - J\boldsymbol{\sigma}\cdot\mathbf{M}(\mathbf{r})\,, \tag{1}$$

where J is the exchange parameter, $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices, and the unit system with $\hbar = 1$ is used.

The domain wall is characterized by a magnetization profile $\mathbf{M}(\mathbf{r})$. For the sake of simplicity we assume $|\mathbf{M}(\mathbf{r})| = M_0 = \text{const}$. Thus, we can write

$$J\mathbf{M}(\mathbf{r}) = M\mathbf{n}(\mathbf{r}), \tag{2}$$

where $\mathbf{n}(\mathbf{r})$ is a unit vector field specific for a particular type of DWs (to be defined later), and $M = JM_0$ is measured in energy units.

In order to control the charge density of the electron gas, we include the Coulomb electron-electron interaction in the mean-field approximation *via* the term

$$H_{int} = e\phi(z)\,, \tag{3}$$

where e is the electron charge ($e < 0$) and the field $\phi(z)$ is the mean-field electrostatic potential in the presence of the wall, which obeys the equation

$$\frac{d^2\phi(z)}{dz^2} = -4\pi e\left(\langle\psi^\dagger\psi\rangle - n_0\right), \tag{4}$$

with $\langle .....\rangle$ denoting the ground state average, n_0 being the electron gas density in the absence of DW, and ψ and $\psi^\dagger$ denoting the spinor field operators. The potential $\phi(z)$ has to be calculated self-consistently, which assures that the total charge accumulated at the wall vanishes, though the charge neutrality may be violated locally. The total Hamiltonian H of the system can be then written as

$$H = H_0 + H_{int}\,, \tag{5}$$

where H_0 and H_{int} are given by Eq.(1) and Eq.(3), respectively.

2.2. *Gauge transformation*

The key point of the approach is a local unitary transformation

$$\psi \to T(\mathbf{r})\,\psi, \quad T^\dagger(\mathbf{r})\,T(\mathbf{r}) = \check{1}, \tag{6}$$

where $\check{1}$ is the 2×2 unit matrix. $T(\mathbf{r})$ transforms the problem of electrons in a system with nonuniform magnetization to an equivalent problem of electrons in a system with uniform magnetization, but with an additional gauge field.[10,18] In other words, $T(\mathbf{r})$ transforms the second term in Eq. (1) as

$$\boldsymbol{\sigma}\cdot\mathbf{n}(\mathbf{r}) \to \sigma_z\,, \tag{7}$$

or equivalently

$$T^\dagger(\mathbf{r})\,\boldsymbol{\sigma}\cdot\mathbf{n}(\mathbf{r})\,T(\mathbf{r}) = \sigma_z\,. \tag{8}$$

Explicit form of $T(\mathbf{r})$ is given by[21]

$$T(\mathbf{r}) = \frac{1}{\sqrt{2}}\left(\check{1}\sqrt{1+n_z(\mathbf{r})} + i\,\frac{n_y(\mathbf{r})\,\sigma_x - n_x(\mathbf{r})\,\sigma_y}{\sqrt{1+n_z(\mathbf{r})}}\right). \tag{9}$$

Generally, the above transformation can be applied not only to simple DWs, but also to other types of topological excitations in ferromagnetic systems, for instance to helicoidal waves, skyrmions, and others.

Upon applying the transformation (6) to the kinetic part of the Hamiltonian (1) one obtains

$$\frac{\partial^2}{\partial\mathbf{r}^2} \to \left(\frac{\partial}{\partial\mathbf{r}} + \mathbf{A}(\mathbf{r})\right)^2, \tag{10}$$

where the non-Abelian gauge field $\mathbf{A}\,(\mathbf{r})$ is given by

$$\mathbf{A}(\mathbf{r}) = T^\dagger(\mathbf{r})\,\frac{\partial}{\partial\mathbf{r}}\,T(\mathbf{r}). \tag{11}$$

According to Eq. (9), the gauge field $\mathbf{A}(\mathbf{r})$ is a matrix in the spin space.

Assume now a more specific DW in a bulk system, which is translationally invariant in the x-y plane: $\mathbf{M}(\mathbf{r}) \to \mathbf{M}(z)$ and $\mathbf{n}(\mathbf{r}) \to \mathbf{n}(z)$. For a simple DW with $\mathbf{M}(z)$ lying in the plane normal to the wall one can parameterize the vector $\mathbf{n}(z)$ as

$$\mathbf{n}(z) = (\,\sin\varphi(z),\; 0,\; \cos\varphi(z)\,)\,, \tag{12}$$

where the phase $\varphi(z)$ determines the type of DWs. The transformation (9) is then reduced to

$$T(z) = \frac{1}{\sqrt{2}}\left(\check{1}\sqrt{1+\cos\varphi(z)} - i\sigma_y\,\frac{\sin\varphi(z)}{\sqrt{1+\cos\varphi(z)}}\right), \tag{13}$$

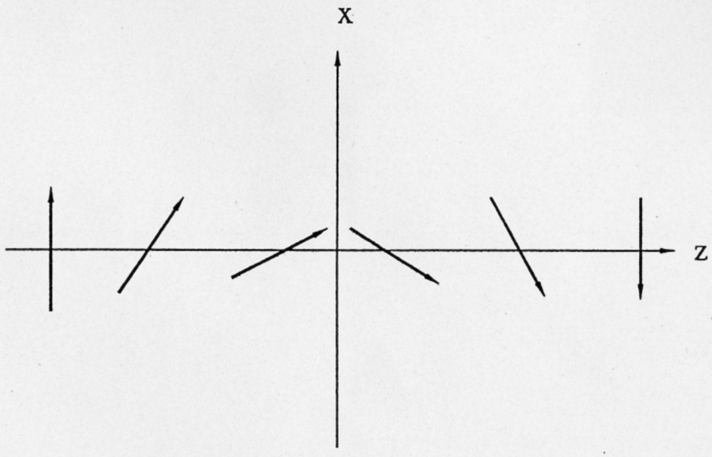

Figure 1. Variation of the magnetization in the domain wall.

and the gauge field acquires the simple form

$$\mathbf{A}(z) = \left(0,\, 0,\, -\frac{i}{2}\,\sigma_y\, \varphi'(z)\right), \tag{14}$$

where $\varphi'(z) \equiv \partial\varphi(z)/\partial z$.

Taking into account the above formulas one can write the full transformed Hamiltonian in the form

$$H = -\frac{1}{2m}\frac{\partial^2}{\partial \mathbf{r}^2} - M\sigma_z + e\phi(z) + \frac{m\beta^2(z)}{2} + i\sigma_y\frac{\beta'(z)}{2} + i\sigma_y\beta(z)\frac{\partial}{\partial z}, \tag{15}$$

where

$$\beta(z) = \frac{\varphi'(z)}{2m}. \tag{16}$$

When $k_{F\uparrow(\downarrow)}D >> 1$, the perturbation due to DW is weak and the semiclassical approximation is well justified. The last three terms on the right hand side of Eq. (15) can be then treated as a small perturbation.

If one assumes the domain wall in the form of a kink shown schematically in Fig. 1, then

$$\varphi(z) = -\frac{\pi}{2}\,\tanh\left(z/L\right) \tag{17}$$

with $L = D/2$, and the parameter $\beta(z)$ is given by

$$\beta(z) = -\frac{\pi}{4mL\,\cosh^2(z/L)}. \tag{18}$$

2.3. *Local conductivity*

The general formula for the local conductivity (without localization corrections and for electric field applied along the axis z) has the following form

$$\sigma_{zz} = \frac{e^2}{2\pi m^2}\,\mathrm{Tr}\int\frac{d^3\mathbf{k}}{(2\pi)^3}(k_z - m\beta\sigma_y)\,G^R_{\mathbf{k}}\,(k_z - m\beta\sigma_y)\,G^A_{\mathbf{k}}, \tag{19}$$

where the gauge potential $\mathbf{A}(z)$ given by Eq. (14) is taken into account, and the retarded (R) and advanced (A) Green functions are both evaluated at the Fermi level,

$$G^{R,A}_{\mathbf{k}} = \frac{-\varepsilon_{\mathbf{k}} - M\sigma_z - k_z\beta(z)\sigma_y + \mu_r(z)}{[-\varepsilon_{\mathbf{k}\uparrow}(z) + \mu_r(z) \pm i/2\tau_\uparrow(z)]\,[-\varepsilon_{\mathbf{k}\downarrow}(z) + \mu_r(z) \pm i/2\tau_\downarrow(z)]}. \tag{20}$$

Here, $\varepsilon_{\mathbf{k}} = (q^2 + k_z^2)/2m$ with $q^2 = k_x^2 + k_y^2$, $\mu_r(z) = \mu - m\,\beta^2(z)/2 - e\phi(z)$ with μ denoting the chemical potential, and

$$\varepsilon_{\mathbf{k}\uparrow(\downarrow)}(z) = \varepsilon_{\mathbf{k}} \mp \left[M^2 + k_z^2\,\beta^2(z)\right]^{1/2}, \tag{21}$$

where the upper (lower) sign refers to $\uparrow$ ($\downarrow$). The quasi-particle energies $\varepsilon_{\mathbf{k}\uparrow(\downarrow)}(z)$ are the eigenstates of the whole Hamiltonian (poles of the Green functions). They correspond to pure spin states only outside the wall, whereas inside the wall they have no pure spin-up (spin-down) form because of spin mixing by the wall. Finally, $\tau_\uparrow(z)$ and $\tau_\downarrow(z)$ in Eq.(20) are the relaxation times, which for impurity scattering potential V_0 independent of the electron spin have the form

$$\frac{1}{\tau_{\uparrow(\downarrow)}(z)} = \frac{m\,V_0^2}{2\pi}\left[k_{F\uparrow}(z) + k_{F\downarrow}(z) \pm \frac{M}{\beta(z)}\,\mathrm{arcsinh}\,\frac{k_{F\uparrow}(z)\,\beta(z)}{M} \mp \right.$$
$$\left. \mp\,\frac{M}{\beta(z)}\,\mathrm{arcsinh}\,\frac{k_{F\downarrow}(z)\,\beta(z)}{M}\right], \tag{22}$$

where $k_{F\uparrow(\downarrow)}(z)$ are the appropriate Fermi wavevectors,

$$k^2_{F\uparrow(\downarrow)}(z) = 2m\mu_r(z) + 2m^2\beta^2(z)\,\pm$$
$$\pm\,2m\left[2m\mu_r(z)\beta^2(z) + m^2\beta^4(z) + M^2\right]^{1/2}. \tag{23}$$

The difference in scattering times is due to a difference in the density of states at the Fermi level for $\uparrow$ and $\downarrow$ states.

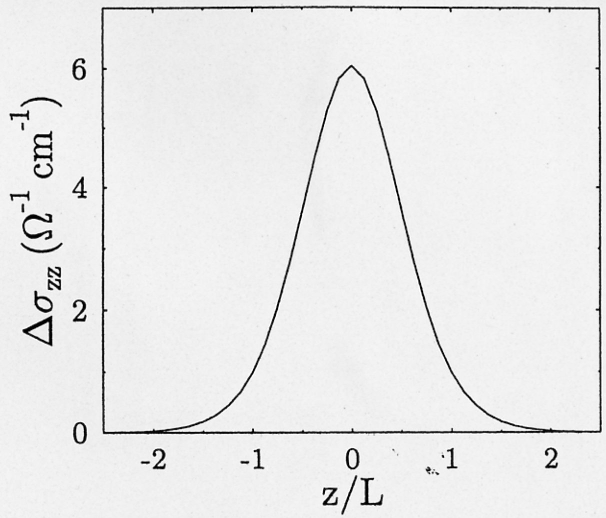

Figure 2. Domain wall contribution to local conductivity, calculated for $L = 50$ Å, Fermi energies $E_{F\uparrow} = 3$ eV and $E_{F\downarrow} = 2.5$ eV, and for impurity scattering potential leading to the bulk conductivity (without domain wall) $\sigma = 0.67 \cdot 10^5$ Ohm^{-1} cm^{-1}.

The local conductivity σ_{zz} is a smoothly varying function of z, $\sigma_{zz} = \sigma_{zz}(z)$,

$$\sigma_{zz}(z) = \frac{e^2}{2\pi^2 m} \sum_{\sigma=\uparrow,\downarrow} \tau_\sigma(z) \left(\frac{k^3_{F\sigma}(z)}{3} + m^2\beta^2(z)\, k_{F\sigma}(z) - \right.$$

$$\left. - m^2 M \beta(z) \arctan \frac{k_{F\sigma}(z)\beta(z)}{M} \right) . \qquad (24)$$

Given the conductivity $\sigma_{zz}(z)$, the average resistivity of a sample of length d with a domain wall can be found as

$$\rho = \frac{1}{d} \int \frac{dz}{\sigma_{zz}(z)} . \qquad (25)$$

The DW contribution to the local conductivity, $\Delta\sigma_{zz}$, is shown in Fig.2 as a function of z. It is evident that this contribution is positive, i.e., the conductivity is enhanced within the wall. The enhancement shown in Fig.2 is not large, but it could be larger when one would assume appropriate spin asymmetry of the impurity scattering potential. It should be noted, however, that by taking opposite spin asymmetry for the impurity scattering potential, the enhancement can be diminished or even can change sign, i.e., the conductivity within the wall can be lower than outside the wall. Thus, this model can account for both signs of the magnetoresistance associated with DWs. This sign depends on the spin asymmetry of impurity scattering potential.

2.4. *Weak localization effects*

It is well known that external magnetic field suppresses the WL corrections to conductivity. More specifically, the vector potential associated with the magnetic field produces an additional phase shift which destroys the quantum interference effects responsible for these corrections. From the discussion of the two preceding subsections follows, that the DW effect on electrons in a ferromagnetic metal can be described in terms of the spin dependent gauge potential $\mathbf{A}(\mathbf{r})$. One may then expect, that this gauge potential has a similar influence on the quantum corrections to conductivity as the vector potential associated with an external magnetic field. This problem was analyzed by Tatara and Fukuyama,[10,14] and also by Lyanda-Geller *et al.*[22]

The quantum corrections to conductivity due to WL are important at low temperatures, while at higher temperatures they are suppressed by inelastic scattering processes. They are usually accounted for theoretically in terms of *cooperons*,[19,20] i.e., propagators which describe propagation in space and time of the superconductive density fluctuations. In nonmagnetic systems there are singlet and triplet *cooperons*, which contribute with opposite signs. There is, however, an essential difference between WL effects in nonmagnetic and ferromagnetic systems, even if there are no DWs in the latter case. This difference is due to internal magnetic field and associated vector potential, which diminishes or even suppresses in some cases the WL corrections in ferromagnets. If, however, the corrections are not suppressed, then the magnetization usually suppresses the singlet *cooperon*. As a consequence, there is no weak antilocalization effect in ferromagnetic systems with strong spin-orbit scattering, contrary to nonmagnetic ones. Accordingly, the magnetoresistance related to WL in ferromagnets is always negative.[23]

The *cooperon* $C(\mathbf{r},\mathbf{r}')$ can be found from relevant ladder-type diagrams.[19,20,22,23] The corresponding localization correction to the local conductivity is then related to the *cooperon via* the expression

$$\delta\sigma = -2D\frac{e^2}{\pi}C(\mathbf{r},\mathbf{r}), \qquad (26)$$

where D is the diffusion constant The key question is then to find the *cooperon* from the appropriate integral or differential equations.

The effect of DW on quantum corrections is related to a specific mechanism of the suppression of cooperons by the gauge field associated with the domain walls. Accordingly, the resistance of a system without DWs is larger than in their presence because DWs destroy the negative corrections to conductivity. Associated magnetoresistance is therefore negative. Weak localization correction to conductivity in ferromagnets in the presence of DWs was analyzed by Tatara and Fukuyama[10,14] as well as by Lyanda-Geller *et al*,[22] but in the case of quasi-one-dimensional wires. When the lateral dimension $d_\perp$ obeys the condition $d_\perp \gg l \gg k_F^{-1}$, then electron motion is three dimensional. However, when additionally $l_\phi \gg d_\perp$, where l_ϕ is the phase coherence length, the system behaves like quasi-one-dimensional from the point of view of WL effects. Under this condition Lyanda-Geller *et al*[22] found the localization correction

$$\delta\sigma^{(1)} = -\frac{e^2}{4\pi}(l_\phi + l_w) \tag{27}$$

where $l_w^{-2} = D^{-1}(\tau_w^{-1} + \tau_\phi^{-1})$. Here τ_ϕ is phase coherence time in the absence of DW, and $1/\tau_w = D/4M^2\tau^2 Ld$. Thus, the wall contributes to phase decoherence and one can note that $\delta\sigma^{(1)}$ in the presence of the wall is smaller than in its absence. Accordingly, conductivity of a system with DW is larger than that of a system without the wall. The parameters τ_ϕ, l_ϕ, and k_F, which enter Eq.(27), were assumed to be independent of the spin orientation. In a general case, however, they are spin dependent.

The above considerations apply to DWs which are uniform in their planes. When, however, this condition is not fulfilled, then there is an additional factor leading to suppression of the WL corrections, which is related to the Berry phase.[22] This contribution is described quantitatively by the additional dephasing length l_M related to the fact that the magnetization (which is coupled to electron spin) encircles a nonzero solid angle Ω for electrons completing a self-crossing trajectory. It is worth to note that $\Omega = 0$ for DWs which are uniform in their planes.

3. Transport through an atomic-size domain wall

3.1. *Scattering states*

Let us consider again the Hamiltonian (1) describing electrons in a spatially inhomogeneous magnetization $\mathbf{M}(\mathbf{r})$. For a very narrow constrained DW one may consider only a few channels for electronic transport. A limiting situation is when there is only a single transport channel. In such a one-dimensional case the Hamiltonian (1) can be rewritten as

$$H = -\frac{1}{2m}\frac{d^2}{dz^2} - JM_z(z)\,\sigma_z - JM_x(z)\,\sigma_x . \tag{28}$$

We will make use of the scattering states taken in the form

$$\chi_{R\uparrow k}(z) = \begin{cases} \begin{pmatrix} e^{ik_\uparrow z} + r_{R\uparrow}\, e^{-ik_\uparrow z} \\ r^f_{R\uparrow}\, e^{-ik_\downarrow z} \end{pmatrix}, & z \ll -L \\ \\ \begin{pmatrix} t_{R\uparrow}\, e^{ik_\downarrow z} \\ t^f_{R\uparrow}\, e^{ik_\uparrow z} \end{pmatrix}, & z \gg L \end{cases} \tag{29}$$

where $k_{\uparrow(\downarrow)} = \sqrt{2m(E \pm M)}$, and E is the electron energy. This state describes the spin-up electron wave incident from $-\infty$ and partly reflected and transmitted into the spin-up and spin-down channels. The coefficients $t_{R\uparrow}$ and $t^f_{R\uparrow}$ are the transmission amplitudes without and with spin reversal, respectively, whereas $r_{R\uparrow}$ and $r^f_{R\uparrow}$ are the relevant reflection amplitudes. The analogous form have the scattering states related to the spin down wave incident from left to right (labeled with $R\downarrow k$), as well as the scattering states related to electron waves incident on DW from the right.

Integrating the Schrödinger equation $H\psi = E\psi$ with the Hamiltonian (28) from $-\delta$ to $+\delta$ in the vicinity of $z = 0$ (where the domain wall is located), and assuming $L \ll \delta \ll k^{-1}_{\uparrow(\downarrow)}$, one obtains

$$-\frac{1}{2m}\left(\left.\frac{d\chi_n}{dz}\right|_{+\delta} - \left.\frac{d\chi_n}{dz}\right|_{-\delta}\right) - \lambda\,\sigma_x\,\chi_n(0) = 0\,, \tag{30}$$

where n is the electron state index ($n \equiv R(L)\uparrow(\downarrow)k$) and λ is a factor defined as

$$\lambda \simeq \int_{-\infty}^{\infty} dz\, JM_x(z) \simeq ML \tag{31}$$

Equation (30) has the form of a spin-dependent condition for transmission through a δ-like potential barrier located at $z = 0$.

Taking into account the scattering states (29) and the condition (30), in combination with the continuity condition for the wave functions, one finds the following expressions for the transmission amplitudes:

$$t_{R\uparrow(\downarrow)} = t_{L\downarrow(\uparrow)} = \frac{2v_{\uparrow(\downarrow)}(v_\uparrow + v_\downarrow)}{(v_\uparrow + v_\downarrow)^2 + 4\lambda^2}, \tag{32}$$

$$t^f_{R\uparrow(\downarrow)} = t^f_{L\downarrow(\uparrow)} = \frac{4i\lambda\, v_{\uparrow(\downarrow)}}{(v_\uparrow + v_\downarrow)^2 + 4\lambda^2}, \tag{33}$$

where $v_{\uparrow(\downarrow)} = k_{\uparrow(\downarrow)}/m$.

According to Eq.(33), the magnitude of the spin-flip transmission coefficient can be estimated as

$$|t^f|^2 \sim \left(\frac{\lambda v}{v^2+\lambda^2}\right)^2 \sim \left(\frac{M\varepsilon_0}{\varepsilon_F\,\varepsilon_0 + M^2}\right)^2 (k_F L)^2, \tag{34}$$

where $\varepsilon_F = k_F^2/2m$, and $\varepsilon_0 = 1/mL^2$. For $k_F L \ll 1$ one finds $\varepsilon_0 \gg \varepsilon_F$. Taking $\varepsilon_F \sim M$, once obtains

$$|t^f|^2 \sim \left(\frac{M}{\varepsilon_F}\, k_F L\right)^2 \ll 1. \tag{35}$$

Thus, a sharp domain wall can be considered as an effective barrier for the spin-flip transmission.

It should be noted that the conservation of flow has the following form

$$v_\uparrow\left(1 - |r_{R\uparrow}|^2\right) - v_\downarrow\left|r^f_{R\uparrow}\right|^2 = v_\downarrow\,|t_{R\uparrow}|^2 + v_\uparrow\left|t^f_{R\uparrow}\right|^2, \tag{36}$$

and analogous equations hold also for the other scattering states.

3.2. *Resistance of the domain wall*

To calculate the conductivity we start from the current operator

$$\hat{j}(z) = e\,\psi^\dagger(z)\,\hat{v}\,\psi(z). \tag{37}$$

Expanding $\psi(z)$ in the scattering states (30), and performing quantum-mechanical averaging, one obtains the following formula for the current

$$j(z) = -ie\sum_n \int \frac{d\varepsilon}{2\pi}\, e^{i\varepsilon\delta}\, G_n(\varepsilon)\, \chi_n^\dagger(z)\,\hat{v}\,\chi_n(z), \tag{38}$$

where n is the index of scattering states. The matrix elements of the velocity operator $\hat{v} = -(i/m)\,\partial_z$ can be calculated in the basis of the scattering states, and one obtains

$$v_{R\uparrow(\downarrow)} \equiv \langle R\uparrow(\downarrow)k\,|\,\hat{v}\,|\,R\uparrow(\downarrow)k\rangle = v_{\downarrow(\uparrow)}\left|t_{R\uparrow(\downarrow)}\right|^2 + v_{\uparrow(\downarrow)}\left|t^f_{R\uparrow(\downarrow)}\right|^2, \tag{39}$$

and similar expressions for the other states.

The retarded Green function $G_n(\varepsilon)$ in Eq. (38) is diagonal in the basis of scattering states. Assuming that the transmission of electrons through the barrier is small, one can take the chemical potential constant $\mu = \mu_R$ for $z < 0$, and $\mu = \mu_L$ for $z > 0$. This corresponds to the voltage drop $U = (\mu_R - \mu_L)/e$ across the barrier. The Green function $G_{R\uparrow k}(\varepsilon)$ acquires then the following simple form

$$G_{R\uparrow k}(\varepsilon) = \frac{1}{\varepsilon - \varepsilon_{R\uparrow}(k) + \mu_R + i\delta}, \tag{40}$$

where $\varepsilon_{R\uparrow}(k) = k^2/2m - M$. The other components of the Green function have a similar form.

After integrating over ε, one finds

$$\begin{aligned} j(z) = e\int \frac{dk}{2\pi}\Big\{ & v_\uparrow\,\chi^\dagger_{R\uparrow k}(z)\,\chi_{R\uparrow k}(z)\,\theta\left[\mu_R - \varepsilon_{R\uparrow}(k)\right] \\ & + v_\downarrow\,\chi^\dagger_{R\downarrow k}(z)\,\chi_{R\downarrow k}(z)\,\theta\left[\mu_R - \varepsilon_{R\downarrow}(k)\right] \\ & - v_\uparrow\,\chi^\dagger_{L\uparrow k}(z)\,\chi_{L\uparrow k}(z)\,\theta\left[\mu_L - \varepsilon_{L\uparrow}(k)\right] \\ & - v_\downarrow\,\chi^\dagger_{L\downarrow k}(z)\,\chi_{L\downarrow k}(z)\,\theta\left[\mu_L - \varepsilon_{L\downarrow}(k)\right]\Big\}. \end{aligned} \tag{41}$$

In view of the conservation of charge, the current does not depend on z, and therefore can be calculated for $z = 0$. Moreover, the total current from the states $\varepsilon_{R\uparrow(\downarrow)}(k), \varepsilon_{L\uparrow(\downarrow)}(k) \le \mu_L$ vanishes and only the states obeying the condition $\mu_L < \varepsilon_{R\uparrow(\downarrow)}(k) < \mu_R$ contribute to the current. The conductance G can be then found as a linear response to small perturbation (in the limit of $U \to 0$), and one finds

$$G = \frac{e^2}{2\pi}\left(\frac{v_\downarrow}{v_\uparrow}\,|t_{R\uparrow}|^2 + \left|t^f_{R\uparrow}\right|^2 + \frac{v_\uparrow}{v_\downarrow}\,|t_{R\downarrow}|^2 + \left|t^f_{R\downarrow}\right|^2\right), \tag{42}$$

where all the velocities and transmission coefficients are taken at the Fermi level.

Finally, using Eqs (32) and (33), one can write the conductance in the form

$$G = \frac{4e^2}{\pi}\,\frac{v_\uparrow\,v_\downarrow\,(v_\uparrow + v_\downarrow)^2 + 2\lambda^2\left(v_\uparrow^2 + v_\downarrow^2\right)}{\left[(v_\uparrow + v_\downarrow)^2 + 4\lambda^2\right]^2} \tag{43}$$

In the limit of $v_\uparrow = v_\downarrow$ and $\lambda \to 0$ one obtains the conductance of a single spin-degenerate channel, $G_0 = e^2/\pi$.

The dependence of G/G_0 on the wall parameter L is shown in Fig. 3 for different values of the parameter M. One can note that the conductance in the presence of a domain wall is generally much smaller than in the absence of the wall. Accordingly, the associated magnetoresistance can be very large (more than 100%, which corresponds to $G/G_0 < 0.5$)), in agreement with experimental observations. It is also worth to note that resistance of an abrupt domain wall is not so large as the resistance of a thicker domain wall (provided the conditions assumed for the model are fulfilled).

4. Summary

We have presented theoretical description of the domain wall contribution to electrical resistivity of metallic ferromagnets. Two limiting cases were

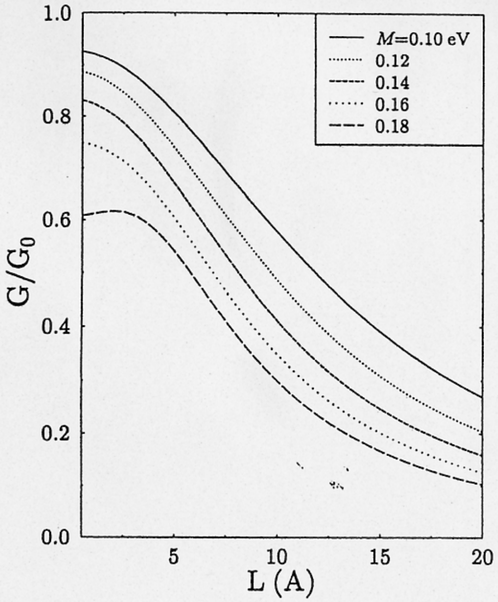

Figure 3. Relative conductance in the presence of a domain wall calculated as a function of L for $E_F = 0.2$ eV and for different values of the parameter M as indicated. For these parameters k_F is about 7 Å.

analyzed in details - the case of a thick domain wall with diffusive electron transport across the wall, and the limit of atomic-size and constrained domain wall, which effectively could be described by a one-dimensional model. These two possibilities are not the only ones. In very pure systems electronic transport across thick domain wall can be ballistic, despite the fact that the domain wall itself may be considered quasi-classically.[24] Apart from this, transport in real nanoconstrictions involves more channels and should be described by a more general theory. Anyway, such an approach may be useful particularly in the cases of point contacts based on new semiconductors heavily doped with magnetic impurities, like ferromagnetic GaMnAs or related compounds.

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