Ultrafast control of electronic motion in quantum-well structures

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An ultrashort half-cycle pulse (HCP) is a fast (<1 ps) unipolar pulse, followed by a much longer (~ 100 ps) and weaker unipolar pulse of opposite polarity. We show that such pulses can be utilized to localize, within femtoseconds, and control, for picoseconds, the electronic motion in a $Al_xGa_{1-x}As$ based symmetric double quantum well. The results are obtained by (i) deriving analytically for a model system the type of HCPs that lead to a fast and sustainable localization of a desirable final electron state and (ii) by solving numerically exactly the time-dependent Schrödinger equation for the quantum-well structure in the presence of the HCPs. © *2004 American Institute of Physics.* [DOI: 10.1063/1.1691191]

Recently there has been an impressive progress in the generation and design of subpicosecond unipolar electromagnetic pulses.[1] The time-dependent electric field in these pulses resembles one half of an optical field cycle in an electromagnetic wave and therefore they are often referred to as ''half-cycle'' pulses (HCPs) [cf. Fig. 1(b)]. For a freely propagating electromagnetic wave Maxwell's equations demand that the time integral over the electric field vanishes. Therefore, a HCP is in fact a strongly asymmetric monocycle pulse consisting of a very short, strong half cycle (only this part is usually referred to as an HCP), followed by a much slower half cycle of an opposite polarity and a much smaller amplitude (called hereafter the tail of the HCP). Typical pulse amplitude asymmetry is 13:1[1] [Fig. 1(b)]. If the round-trip time of a confined electron is much longer that the duration of the HCP, the electron-HCP interaction can be viewed classically as an impulsive ''kick'' received by the electron.[2] The amount of the kick (momentum change) is given by $\mathbf{\Delta p}=-\int \mathbf{F}(t)dt$, where $\mathbf{F}(t)$ is the time-dependent field of the HCP. Quantum mechanically, subjecting the electron to a HCP, results in a linear transformation of the momentum space wave function in the direction of the kick, $\tilde{\Psi}(\mathbf{p})\rightarrow\tilde{\Psi}(\mathbf{p}+\mathbf{\Delta p})$. In the configuration space, applying a HCP phase shifts the electron wave function as $\Psi(\mathbf{r})\rightarrow\Psi(\mathbf{r})e^{-i\mathbf{\Delta p}\cdot\mathbf{r}}$. The weak tail of the HCP acts as an offset dc field that hardly affects the electron dynamics. From this scenario of the electron-HCP interaction one may expect that the position and the momentum of a given electronic distribution can be, to a certain degree, controlled and manipulated by applying a sequence of kicks [cf. Fig. 1(b)] with appropriate relative strengths, delays, and directions. The feasibility of generating such HCPs has been demonstrated.[3]

It is the aim of this work to determine the right properties of a train of HCPs that allows a coherent control, on the *subpicosecond* scale, of the electronic motion in an $Al_xGa_{1-x}As$ based double quantum well. Such a possibility is desirable technologically, e.g., for the design of ultrafast switches or for the construction and control of quantum logic states (e.g., one can associate 1 and 0 with the states in which the electron is localized in the left and in the right well, respectively).

The possibility of controlling quantum coherence in double-well potentials[4–6] and in two-level (TL) systems[4,6,7] has been theoretically explored in considerable detail. Previous studies are limited, however, to the case of a continuous wave (cw) driving laser field. Here we report the investigation on the control of electron quantum dynamics with the aid of a train of ultrashort half-cycle pulses. The nature of the interaction of cw lasers and HCPs with electrons is qualitatively different. In the case of a monochromatic cw laser only the characteristic frequency of the laser is relevant, and no direct transfer of momentum to the electron takes place. In contrast, a HCP has a wide spectrum extending to zero frequency and it can transfer energy and momentum to a bound or free electron.

In view of these differences and the recent achievements in the technology of HCPs[8,9] it is timely to address the coherent manipulation of electronic motion via HCPs. As shown below, an appropriately designed train of HCPs allows the control of electronic motion on the subpicosecond scale, whereas such a process lasts several picoseconds when cw lasers are used.[7] A further important favorable feature of using HCP is that the control process is robust to considerable changes of the field parameters, whereas in the case of cw lasers the control process is a resonance phenomena that is very sensitive to changes in the driving laser field properties. In this context we mention, that nowadays optical-pulse shapers enable fine control over the spectral amplitude and phase of broadband laser pulses,[10] nevertheless the class of

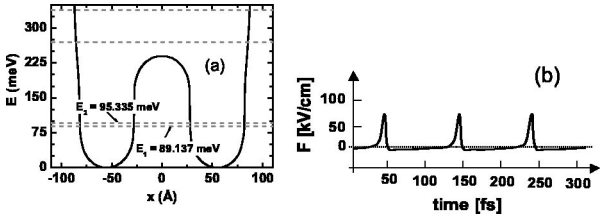

FIG. 1. (a) Electron confining potential: the central barrier hight is ~ 240 meV. The wells and barrier widths are ~ 50 and ~ 60 Å, respectively. Dashed lines indicate the first lowest energy levels. (b) The electric field amplitude vs time for a typical sequence of HCPs.

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wave packets achievable by optical frequency fields is still quite limited.[11] We also remark that the high asymmetry of the HCP is essential for the nonzero transfer of momentum on a time scale much shorter than the characteristic time of the field-free system and therefore cw lasers or nearly symmetric laser pulses are not expected to produce the effects discussed bellow.

We consider a conduction electron confined in a typical $Al_xGa_{1-x}As$ based double quantum well depicted in Fig. 1(a). Within the parabolic band and the effective mass approximations (the effective mass $m^*=0.067m_0$ is assumed constant throughout the heterostructure), the Hamiltonian describing the system is

$$H(t)=H_0+V_{\text{conf}}+V(x,t), \tag{1}$$

where H_0 is the bare Hamiltonian, V_{conf} refers to the confinement potential [Fig. 1(a)], and $V(x,t)$ stands for the coupling of the electron to the pulses. As shown below, a typical localization time is ~ 132 fs which is well below the typical time scale (several picoseconds) for the elastic scattering and electron-phonon interaction in high quality Ga(Al)As–GaAs heterostructures with typical electron concentration.[12] Therefore, these effects are subsidiary for the localization process. The electron coupling to the sequence of (Gaussian-like shaped) HCPs is modeled by the potential $V(x,t)$ which has the form

$$V(x,t)=x\sum_{k=0}^{N-1}\left(F_k\exp\left[-\frac{(t-t_0-kT)^2}{2\sigma^2}\right]\right), \tag{2}$$

where σ characterizes the width of the pulses, t_0 corresponds to the time at which the first applied pulse is centered, T is the time between consecutive pulses, N is the number of applied pulses, and F_k is related to the amplitude of the kth pulse. We checked numerically that including the negative tails of the pulses (as described above) has no visible effect on the results presented below. The time-dependent Schrödinger equation with the Hamiltonian (1) and the driving potential (2) has to be solved numerically. To this end we implemented a fast Fourier transform based numerical method[13] for the time propagation of the initial wave function. Having determined the time-dependent wave function $\Psi(x,t)$, we calculate the time-dependent probability $[P_L(t)=\int_{-\infty}^{0}\Psi^*(x,t)\Psi(x,t)dx]$ and the time averaged probability $[\langle P_L\rangle_\tau=1/\tau\int_0^\tau P_L(t)dt]$ of finding the electron in the left well. All the calculations were performed with $\sigma=10$ fs, the first pulse was centered at $t_0=40$ fs, and the period was taken as $T=100$ fs.

The question of interest here is, what are the values F_k,σ,T and N in Eq. (2) that lead to maximal, sustainable localization $P_L(t)$. In view of the number of these parameters it is impracticable (and of a limited validity) to answer this question via brute numerical simulations. Analytical considerations are due for guiding the numerics. To this end we remark that the two lowest-energy levels are well separated from the other energy states [cf. Fig. 1(a)]. Hence, in a first step we reduce the system to a two-level (TL) problem. The resulting time-dependent Schrödinger equation including Eq. (2) is still not amenable to analytical solutions. Further simplification is brought about by the fact that for ultrashort HCPs the duration of each pulse is much smaller

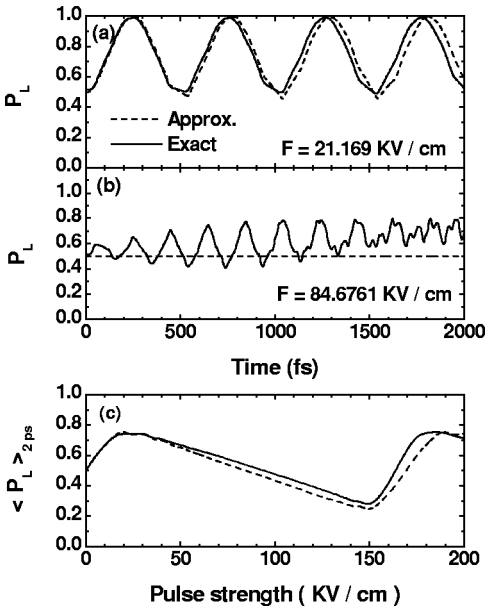

FIG. 2. (a) Time dependence of P_L for a pulse amplitude corresponding to $n=0$ in the localization condition Eq. (3). (b) Same as in (a) but for a pulse amplitude corresponding to $n=1$ [delocalization condition (4)]. (c) P_L averaged over 2 ps vs pulse strength.

than the typical characteristic time T_c of the un-driven system [in our case, $T_c\approx 665$ fs which is much larger than the duration of the pulses (~ 80 fs)]. Hence, one can approximate Eq. (2) by its limit at $\sigma\to 0$,[2] i.e., $V(t)\approx\Sigma_{k=0}^{N-1}[\Delta p_k\delta(t-t_0-kT)]$, where $\delta(x)$ is the Dirac function and $\Delta p_k=F_k\sigma\sqrt{2\pi}$. With this approximation analytical expressions can be deduced that describe exactly the TL driven system (further details are given elsewhere).

Before applying the pulses, the particle is completely delocalized across the heterostructure. For localizing the wave function in the left well the analytical TL model delivers the following parameters for the appropriate pulse

$$\frac{\mu\Delta p}{\hbar}=(2n+1)\frac{\pi}{4}+(-1)^{n+1}\frac{\pi}{8};\quad n\in\mathcal{Z}, \tag{3}$$

where $\Delta p=\Delta p_k$ $(\forall k)$ and the time delay between consecutive pulses is $T<T_c/4$. The term μ is the transition dipole between the two lowest field-free levels. The analytical TL model also predicts complete delocalization (in presence of the pulses) when

$$\frac{\mu\Delta p}{\hbar}=n\frac{\pi}{2};\quad n\in\mathcal{Z}. \tag{4}$$

The exact (numerical) and approximate (TL analytical) time dependence of the probability of finding the electron in the left well is shown in Figs. 2(a) and 2(b) for a pulse sequence obeying Eqs. (3) and (4), respectively. In Fig. 2(c) the average probability $\langle P_L\rangle_{2\text{ps}}$ as a function of the pulse amplitude is displayed. An important conclusion from Fig. 2(c) is that, the localized electron can be steered to one of the wells by choosing an appropriate value for the pulse amplitude. For example, pulse amplitudes corresponding to $n=0$ in Eq. (3) or $n=1$ in Eq. (4) lead to an electron localization in the left or right well, respectively. A similar effect is achieved by changing the direction of the pulses.

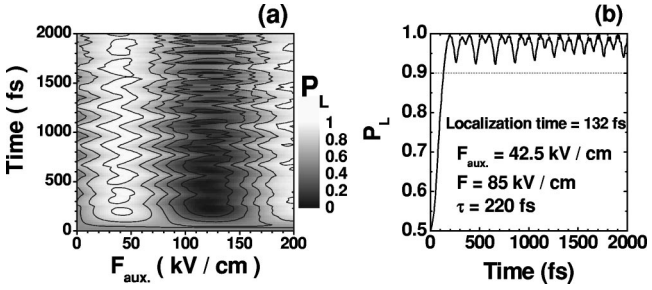

FIG. 3. (a) P_L vs time and peak amplitude F_{aux} of the auxiliary pulse. Following F_{aux} we apply, after a time delay $\tau=220$ fs, a quasiperiodic train of HCPs with peak amplitudes $F_k=85$ kV/cm. (b) A cut in (a) at $F_{\mathrm{aux}}=42.5$ kV/cm.

The localization is further enhanced by applying at first an auxiliary HCP of a strength F_{aux} and, after an appropriate time delay, a quasiperiodic train of HCPs. In effect, the first pulse pushes the electron into the left well and the subsequent train of HCPs keeps the particle localized in that well by kicking it back at the time when it starts tunneling to the second well. For this scenario the analytical TL model delivers the conditions: F_{aux} must be such that $\mu\Delta p_{\mathrm{aux}}/\hbar=\pi/4$; after a time delay $\tau=T_c/4+\gamma$ ($\gamma<T_c/4$) one must apply a train of HCPs with a period $T\approx 2\gamma$ and obeying the condition $\mu\Delta p/\hbar=(2n+1)\,\pi/2$; $n\in\mathcal{Z}$. According to these predictions we performed the exact numerical calculations including the complete spectrum of the system. The results shown in Fig. 3 evidence that strong localization of the initially delocalized electron can be achieved in times of the order of 100 fs. This finding is in sharp contrast to the case when cw lasers are used as driving fields,[5] where it has not been possible to achieve such a strong localization and, in addition, the time needed to achieve electron localization was found to be on the order of few picoseconds.[5] Furthermore, as is clear from Fig. 3(a) the localization is robust to considerable changes in the field strength which makes the present control scheme a good candidate for applications, such as the design of electro-optical devices and ultrafast switches.

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