Topological Hall Effect and Berry Phase in Magnetic Nanostructures

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We discuss the anomalous Hall effect in a two-dimensional electron gas subject to a spatially varying magnetization. This topological Hall effect does not require any spin-orbit coupling and arises solely from Berry phase acquired by an electron moving in a smoothly varying magnetization. We propose an experiment with a structure containing 2D electrons or holes of diluted magnetic semiconductor subject to the stray field of a lattice of magnetic nanocylinders. The striking behavior predicted for such a system (of which all relevant parameters are well known) allows one to observe unambiguously the topological Hall effect and to distinguish it from other mechanisms.

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After half a century of theoretical efforts, the Hall effect of ferromagnets (usually called *anomalous* or *extraordinary* Hall effect) remains a puzzling and controversial topic. Until recently, it was considered that it originates from the combined effect of exchange and spin-orbit (SO) interactions. Two mechanisms of anomalous Hall effect (AHE) have been identified (the *skew scattering* [1,2] and *side jump* [3]) and studied thoroughly [4]. Recently, a new point of view has been proposed [5], in which the AHE is expressed in terms of a Berry curvature in momentum space. However, one should note that a generally accepted theory treating on an equal footing all the above mentioned contributions to the SO-induced AHE is still missing.

Recently, it has been suggested that (in addition to the above mentioned SO-induced mechanism) a new mechanism may give rise to a nonvanishing Hall effect in ferromagnets having a topologically nontrivial (chiral) spin texture, such as manganites or pyrochlore-type compounds [6]. To distinguish this mechanism from the SO-based mechanism, we shall refer to it hereafter as the *topological Hall effect* (THE).

Several theoretical papers have been devoted to the THE. In order to explain the AHE observed in manganites, a model of 3D ferromagnet with thermally excited Skyrmion strings (topological dipoles) has been proposed [7], showing that a THE can be induced by the Berry phase [8] related to the spatial variation of magnetization in the vicinity of the string. The case of disordered ferromagnets in the limit of small exchange splitting has been addressed in Ref. [9]. In both cases, in order to get a net topological field (or chirality), the SO coupling must be invoked. It should be noted that in cases such as the 2D Kagomé lattice or a 3D pyrochlore lattice the net topological field vanishes; a nonvanishing THE may nevertheless be obtained [10], but the sign and strength of the effect depends in a complicated manner on fine details of the magnetic and electronic structures, which makes a quantitative experimental check of the theory questionable. In this respect it would be desirable to have systems having a net topological field that can be controlled by means of some external parameter.

In addition, all the above mentioned discussions of the THE concern systems with spin chirality at the microscopic scale (e.g., pyrochlore lattice) or due to Skyrmion strings. In both cases, quantitative experimental information on the chirality is not easily available. Furthermore, the SO mechanism is usually also present, which complicates the quantitative interpretation of the observations.

In the present Letter, we propose to investigate the THE in nanostructures, namely, in a 2D electron (or hole) gas, in which a well-controlled artificial chirality can be induced from the stray field of a lattice of magnetic nanocylinders. The great advantage of such a model system is that, in contrast to the above mentioned cases, all relevant parameters are well known and (to some extent) adjustable. We show that this yields a significant net topological field (equivalent to a magnetic field of about 5 kG), and that the latter has a nontrivial characteristic variation with respect to a (uniform) external magnetic field, providing an unambiguous signature of the THE. Finally, we suggest a system appropriate for an experimental check of our theory.

We start from a model of two-dimensional electron gas (2DEG) in a smoothly varying magnetization $\mathbf{M}(\mathbf{r})$. The Hamiltonian has the following form

$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial \mathbf{r}^2} - g\boldsymbol{\sigma}\cdot\mathbf{M}(\mathbf{r}), \tag{1}$$

where g is the coupling constant and $\boldsymbol{\sigma}$ is the vector of Pauli matrices. We assume that the amplitude of magnetization is constant, $\mathbf{M}(\mathbf{r}) = M\mathbf{n}(\mathbf{r})$, and that the 3D unit vector $\mathbf{n}(\mathbf{r})$ is a slowly varying function of coordinates.

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We use a gauge transformation $T(\mathbf{r})$, which makes the quantization axis oriented along vector $\mathbf{n}(\mathbf{r})$ at each point [11]. It transforms the last term in (1) as $T^{\dagger}(\mathbf{r}) \times [\boldsymbol{\sigma} \cdot \mathbf{n}(\mathbf{r})] T(\mathbf{r}) = \sigma_z$, corresponding to a local rotation of the quantization axis from z axis to the axis along $\mathbf{n}(\mathbf{r})$.

The transformed Hamiltonian describes the electrons moving in a (spinor) gauge potential $\mathbf{A}(\mathbf{r})$:

$$H' = -\frac{\hbar^2}{2m}\left[\frac{\partial}{\partial \mathbf{r}} - \frac{ie}{\hbar c}\mathbf{A}(\mathbf{r})\right]^2 - gM\sigma_z, \tag{2}$$

where $A_i(\mathbf{r}) = -2\pi i \phi_0 T^{\dagger}(\mathbf{r}) \partial_i T(\mathbf{r})$, $\phi_0 = hc/|e|$ is the flux quantum, and $i = x, y$. For convenience, we define the gauge potential $\mathbf{A}(\mathbf{r})$ to have the same dimension as the electromagnetic vector potential. The components of $\mathbf{A}(\mathbf{r})$ can be found easily using an explicit form of $T(\mathbf{r})$.

Hamiltonian (2) with the matrix $\mathbf{A}(\mathbf{r})$ contains terms inducing transitions between the spin-polarized states. We consider the case when spin-flip transitions can be neglected, i.e., where the spin adiabatically follows $\mathbf{n}(\mathbf{r})$. It corresponds to the condition $\lambda \equiv (\varepsilon_F/\varepsilon_0)(k_F\xi)^{-1} \ll 1$, where ξ is a characteristic length of the variation of $\mathbf{n}(\mathbf{r})$, ε_F is the Fermi energy, and $\varepsilon_0 = 2gM$ is the spin splitting.

We also assume that the 2DEG is half metallic with the Fermi level located in the spin-up subband. Then we can neglect the spin-down electrons, and we obtain the following effective Hamiltonian of spinless electrons:

$$\tilde{H} = -\frac{\hbar^2}{2m}\left[\frac{\partial}{\partial \mathbf{r}} - \frac{ie}{\hbar c}\mathbf{a}(\mathbf{r})\right]^2 + V(\mathbf{r}), \tag{3}$$

where

$$a_i(\mathbf{r}) = \frac{\pi\phi_0(n_x\partial_i n_y - n_y\partial_i n_x)}{1 + n_z}, \tag{4}$$

the potential $V(\mathbf{r}) = (\hbar^2/8m)(\partial_i n_\mu)^2$ results from the second order in $\mathbf{A}(\mathbf{r})$ terms, and $\mu = x, y, z$. For spin-down electrons the sign of the gauge field $\mathbf{a}(\mathbf{r})$ in Eq. (3) is reversed.

The topological field is defined as $B_t = \partial_x a_y - \partial_y a_x$. It acts on the electrons within the spin-polarized subband like the ordinary magnetic field, and, in particular, gives rise to a Lorentz-type force [12]. Using (4) we find

$$B_t = \frac{\phi_0}{4\pi}\epsilon_{\mu\nu\lambda} n_\mu (\partial_x n_\nu)(\partial_y n_\lambda), \tag{5}$$

where $\epsilon_{\mu\nu\lambda}$ is the unit antisymmetric tensor. The integral of (5) over an area S_0 enclosed by an arbitrary contour L_0 is proportional to the Berry phase calculated as the spherical angle spanned by area S inside the contour L in $\mathbf{n}$ space. This results from the mapping of L_0 onto the contour L in the mapping space S_2. In the 2D case with a constant magnetization at infinity, we can compactify 2D plane to a sphere S_2, and the integral of (5) over S_2 is proportional to topological invariant corresponding to the number of covering the mapping space.

We consider a mesoscopic system consisting of a 2DEG subject to the stray field of a periodic 2D lattice of magnetic nanocylinders [13] (uniformly magnetized along the z axis), as sketched on Fig. 1. The stray field induced in the 2DEG equals zero on average, but its spacial fluctuations induce in the 2DEG a topological field with a nonzero average. The topological field distribution is solely determined by a few geometric parameters such as the lattice type, cylinder diameter, intercylinder spacing, and the distance between the 2DEG and the bottom of the cylinders. In any practical realization, all these parameters (and hence the topological field) are well known and (to some extent) easily adjustable. From the discussion following Eq. (5), one concludes immediately that the net topological flux per unit cell is equal to an integer multiple of ϕ_0, i.e., the average topological field is $\overline{B}_t = I\phi_0/s_0$, where I is an integer and s_0 the unit cell area. Using the semiclassical Drude theory of magnetotransport (which is valid in the weak field limit) and using the mean field approximation (i.e., neglecting spacial fluctuations of the topological and magnetic stray fields), one obtains $\sigma_{xx} = \sigma^{\uparrow}_{xx} + \sigma^{\downarrow}_{xx}$ and $\sigma_{xy} = \sigma^{\uparrow}_{xy} + \sigma^{\downarrow}_{xy}$ (since the two spin channels are decoupled in the rotated spin-frame) with:

$$\sigma^s_{xx} = \frac{n_s e^2 \tau_s}{m}, \tag{6}$$

$$\sigma^s_{xy} = s\sigma^s_{xx}\frac{e\overline{B}_t\tau_s}{m}, \tag{7}$$

where $s = +1$ (respectively, $s = -1$) for $\uparrow$ spin (resp. $\downarrow$ spin).

We have calculated the stray field and topological field for a triangular lattice of nanocylinders (of magnetization M_s along the z axis), using realistic values of parameters for the nanocylinder lattice [13]. Namely, we take a lattice constant of 100 nm, a cylinder radius of 37 nm (i.e., a filling ratio of 50%), and the gap D between the lattice and semiconductor equal to 20 nm. The distribu-

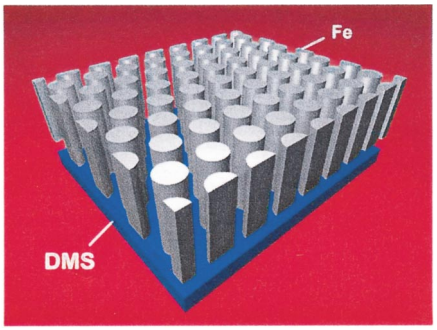

FIG. 1 (color). The proposed structure consisting of a triangular lattice of magnetic nanocylinders on top of 2D diluted magnetic semiconductor.

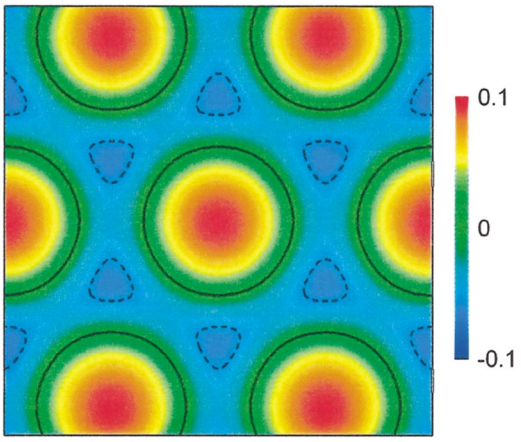

FIG. 2 (color). Distribution of the z-component of dipolar field $B/4\pi M_s$ inside the semiconductor film for the triangular lattice of magnetic nanocylinders, for a zero external field. The black solid circles correspond to the lines with $B_z = 0$. Dashed lines correspond to the lines with $B_z = 0$ under a uniform external magnetic field $B_{\text{ext}}/4\pi M_s = +0.058$.

tion of z component of the dipolar field (normal to the semiconductor surface) is shown in Fig. 2. The corresponding topological field calculated from Eq. (5) is presented in Fig. 3. For Fe cylinders, the dipolar field B inside the semiconductor is about 2 kG in absolute value. The characteristic length for the variations of $\mathbf{n}(\mathbf{r})$ is of the order of the gap D, i.e., $\xi \simeq 20$ nm.

One can see quite easily (for example by considering the Berry phase from the mapping of the plane onto the sphere S_2) that the net topological flux per unit cell ϕ_t has to be an integer multiple of ϕ_0, and that ϕ_t/ϕ_0 is given by the number (per unit cell) of lines $B_z = 0$ enclosing a region with $B_z < 0$ minus the number of lines $B_z = 0$ enclosing a region with $B_z > 0$. From Fig. 2, one thus sees that we have a triangular lattice of lines $B_z = 0$ enclosing regions with $B_z < 0$, yielding $\phi_t = \phi_0$ for a vanishing external field. For the considered geometry this corresponds to an average topological field of about 5 kG, with local values ranging between -5 kG and $+15$ kG.

If a negative external field is applied (assuming that the magnetization of the nanocylinders remains unchanged, due to some large coercivity), the lines with $B_z = 0$ shrink without changing their topology until they collapse to a single point and eventually disappear at a critical field B_1, beyond which $\phi_t = 0$. If a positive field is applied, the lines with $B_z = 0$ expand, until they connect each other at a critical field B_2 and change their topology to a honeycomb lattice (the dual of the triangular lattice) of lines $B_z = 0$ enclosing regions with $B_z > 0$ (dashed lines in Fig. 2), giving $\phi_t = -2\phi_0$. Increasing further the external field leads to a collapse of the lines with $B_z = 0$ at a critical field B_3, beyond which $\phi_t = 0$. For Fe nanocylinders in the above geometry, the critical fields are, respectively, $B_1 \simeq -2$ kG, $B_2 \simeq +0.9$ kG, and $B_3 \simeq +1.3$ kG. This change of topology under application of an external magnetic field results in a striking field dependence of the Hall conductivity, as sketched in Fig. 4, and constitutes an unambiguous signature of the THE. Note that, in any realistic case, the adiabaticity condition would not be satisfied in the vicinity of the critical fields $B_{1,2,3}$, leading to a "rounding" of the jumps in the Hall conductivity.

For the practical realization, we propose to use II–VI dilute magnetic semiconductors (DMS) which exhibit giant Zeeman splitting; p-type DMS are best suited since the exchange constants for holes are much larger than for electrons [14]. For the estimation of the topological field acting on holes, we use the Luttinger Hamiltonian [15]

$$H_h = \frac{\hbar^2}{2m_0}\left[\left(\gamma_1 + \frac{5}{2}\gamma_2\right)\nabla^2 - 2\gamma_2(\mathbf{J}\cdot\nabla)^2\right] + E_{ex}\mathbf{J}\cdot\mathbf{n}(\mathbf{r}), \tag{8}$$

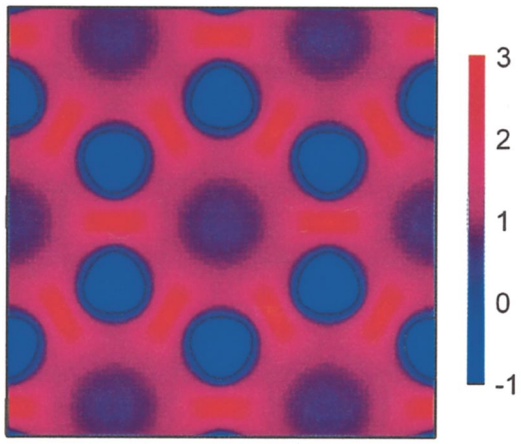

FIG. 3 (color). Topological field $B_t(\mathbf{r})$ (in units of ϕ_0 per unit cell area) for the triangular lattice of magnetic nanocylinders. Black lines correspond to $B_t = 0$.

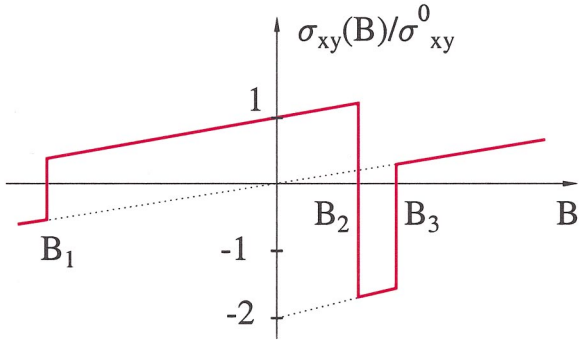

FIG. 4 (color). Dependence of the Hall conductivity on external magnetic field (schematically). The slope corresponds to the contribution of the normal Hall effect; σ^0_{xy} is the Hall conductivity corresponding to a topological flux per unit cell equal to ϕ_0.

where J_μ are the matrices of momentum $J = 3/2$, γ_1 and γ_2 are the Luttinger parameters, m_0 is the free electron mass, $\mathbf{n}(\mathbf{r})$ is the unit vector along the direction of exchange field acting on 2D gas of holes, E_{ex} is the average exchange field created by magnetic impurities,

$$E_{ex} = -JSxN_0\beta\mathcal{B}_S(g_L\mu_B B/k_B T), \tag{9}$$

$N_0\beta$ is the $p-d$ exchange constant, S is the magnetic moment of impurity, xN_0 is the concentration of magnetic atoms, $\mathcal{B}_S(z)$ is the modified Brillouin function, and g_L is the Landé factor of magnetic atom.

We calculate the topological field for holes described by Hamiltonian (8), using the transformation $T(\mathbf{r}) = \exp[i\mathbf{J}\cdot\mathbf{\Omega}(\mathbf{r})/2]$, where $\mathbf{\Omega}(\mathbf{r})$ is the vector of rotation. We find the topological field B_t considering small deviations of the vector $\mathbf{n}(\mathbf{r})$ in a vicinity of some point $\mathbf{r}$. Assuming that the magnetic splitting under exchange field is strong, we find that the gauge potential and the topological field for holes in the valence band, being proportional to J, are given by the same expressions, (4) and (5), as for electrons, multiplied by a factor 3.

After transforming (8) and restricting ourselves by the subband $J_z = -3/2$, we obtain for spin-polarized holes, the same Hamiltonian as (3), with $-1/m$ replaced by $1/m^\star = (\gamma_1 + \gamma_2)/m_0$.

The spin splitting in II–VI semiconductor can be large enough to provide 100% polarization of holes under the dipolar field of 2 kG. To show this we make the following estimations, taking the exchange coupling $N_0\beta = -1.2$ eV, which is the typical magnitude for different compounds ($Cd_{1-x}Mn_xSe$, $Zn_{1-x}Mn_xSe$, etc.), and the atomic density of magnetic atoms $x = 0.05$. For $B = 2$ kG, $g_L = 2$, and $T = 4.2$ K the Brillouin function $\mathcal{B}_{5/2}(g\mu_B B/k_B T) \simeq 0.075$. Then using (8) we obtain $E_{ex} \simeq 11$ meV . The Fermi energy of 2D holes in the spin-splitted subband is determined by the density of holes n_p, $E_F = 2\pi\hbar^2 n_p/m^*$, where m^* is the effective mass of holes. By taking $m^* = 0.5m_0$, and $n_p = 10^{11}$ cm^{-2}, we obtain $E_F \simeq 1$ meV. Thus, the condition of 100% polarization of holes, $E_F < E_{ex}$, is quite realistic for II–VI diluted magnetic semiconductors. The adiabaticity parameter for this choice of parameters is $\lambda \simeq 0.04 \ll 1$. It should be noted that much stronger splitting can be reached for $Zn_{1-x}Cr_xTe$ compounds, for which $N_0\beta = 3.6$ eV [16].

For electrons, on the other hand, in order to fulfill the conditions of full polarization and adiabaticity, lower temperatures and/or very low electron densities would be needed. Taking the exchange constant of $s-d$ interaction $N_0\alpha = 0.22$ eV [14], the electron effective mass $m_e = 0.22m_0$, $T = 4.2$ K, and the concentration of electrons $n_e = 10^{11}$ cm^{-2}, we obtain $E_F \simeq 4.4$ meV, $E_{ex} \simeq 2$ meV, and $\lambda \simeq 1$; the condition of adiabaticity is poorly fulfilled, resulting in a reduction of the effect. Also the gas is not fully polarized, reducing further the THE due to the partial compensation of contributions from spin-up and -down subbands. Although not fully developed, the THE should nevertheless be observable with electrons.

In conclusion, we have proposed that the THE can be observed in suitably chosen nanostructures, and that its striking behavior under an external magnetic field provides an unambiguous experimental signature of the THE.

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