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Vacuum fluctuations and spin current in mesoscopic structures with collinear magnetic order

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We show that in magnetic nanostructures with homogeneous magnetic order, the equilibrium spin current can be nonzero. For example, this is the case for a wide magnetic ring with the magnetization along the ring axis. The physical reason for this effect is a variation of the orientation of the anisotropy axis, inducing a spin torque acting on the magnetic ions. The mechanism of the spin current generation is related to the quantum vacuum fluctuations in the magnetic system.

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One of the key problems in modern magnetoelectronics is related to the creation and manipulation of spin currents.[1–4] It was shown recently that a spin current can be generated using the flux of magnons in a nonequilibrium state of the magnetic system.[5] This idea was further developed for magnons in textured magnetic structures like magnetic rings in an inhomogeneous external field.[6,7] In the latter case, the existence of spin currents is related to the gauge field for the motion of magnons.[8,9] The gauge field and, correspondingly, the Berry phase for the adiabatic motion of magnons appear naturally due to the nonvanishing chirality of the magnetic ordering. In terms of the Berry phase, the magnon mechanism of the spin current excitation can be better understood for various types of magnetic structure.[8]

In the case of magnetic systems with topological excitations (like vortices or skyrmions) or a system subject to inhomogeneous fields, there appears an equilibrium spin current associated with the transmission of angular momentum.[9] This current transfers the torque acting on the disoriented magnetic moments. The equilibrium spin current is usually much stronger than the one transferred by magnons because it does not require any spin wave excitations. This mechanism of spin current generation can be realized only in textured ferromagnets, and it disappears in the case of homogeneous magnetization.

In real magnetic structures, one should also take into account a possible inhomogeneity with respect to the orientation of the anisotropy axes. It induces a new type of *topological* Berry phase, which does not necessarily vanish when the usual geometric phase is exactly zero.[10] An example of such a system is a wide magnetic ring (thin-wall cylinder) with the magnetization along the cylinder axis.[8,10] In this system, the gauge field is related to the variation of the anisotropy axis (perpendicular to the cylinder surface).

We show below that the equilibrium spin current does not vanish in a system with homogeneous magnetization but inhomogeneous anisotropy. This effect has a purely quantum origin and is related to the vacuum fluctuations in the magnetic system. The quantum character of this effect makes it substantially different from the earlier studied mechanism of the equilibrium spin current in noncollinear ferromagnets,[9,11] which can be understood within a model of the classical moment.

The spin current appearing in a shaped mesoscopic structure (e.g., a nanoring) with collinear magnetic ordering transfers angular momentum. The resulting transferred torque acts on the anisotropy axes, tending to arrange them homogeneously. In other words, the spin current produces a mechanical stress in an anisotropy-noncollinear system.

The physics of this phenomenon can be better understood if we consider first a simple model with two quantum spins $S=1$ in a magnetic field B directed along the axis z, and with the anisotropy axes oriented along $\mathbf{n}_1$ and $\mathbf{n}_2$ in the x-y plane for spins S_1 and S_2, respectively (Fig. 1). We take $\mathbf{n}_1$ along x and denote the angle between $\mathbf{n}_1$ and $\mathbf{n}_2$ by ϕ. The Hamiltonian is

$$H=-BS_{1z}+\frac{\lambda}{2}(\mathbf{n}_1\cdot\mathbf{S}_1)^2-BS_{2z}+\frac{\lambda}{2}(\mathbf{n}_2\cdot\mathbf{S}_2)^2-g\mathbf{S}_1\cdot\mathbf{S}_2, \tag{1}$$

where $\lambda>0$ is the constant of anisotropy, and the last term corresponds to the exchange interaction with coupling constant g. We assume $g>0$, which corresponds to ferromagnetic coupling. Obviously, in the ground state both spins are oriented along z.

We can use the basis of eigenfunctions of Hamiltonian (1) with $g=0$. For each of the noninteracting spins S_1 and S_2, the eigenfunctions can be easily found, and they differ from each other by the transformation $\exp(i\phi J_z)$, where J_z is the generator of z rotations. In the absence of interaction, $g=0$, the

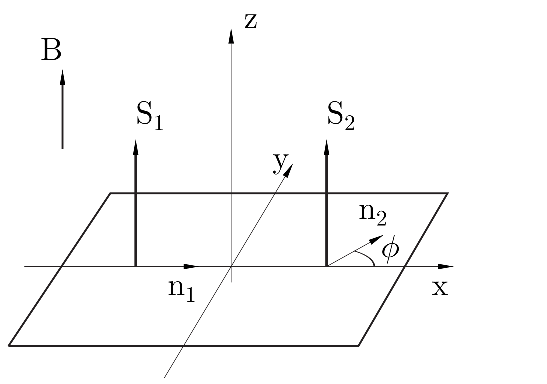

FIG. 1. Two spins $S=1$ with different orientations of anisotropy axes in the x-y plane.

TH-2007-13

lowest four states of Hamiltonian (1) correspond to energies $(2\varepsilon_0, \varepsilon_0+\lambda/2, \lambda)$, where the level $\varepsilon_0+\lambda/2$ is twofold degenerate. We denote these states $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$. Here $\varepsilon_0=\lambda/4-(\lambda^2/16+B^2)^{1/2}$ is the ground state energy of a single spin.

The essential point is that each of the two separated quantum spins in the ground state has the magnitude of the moment along the z axis smaller than unity, $|\langle S_{1z}\rangle_0|=|\langle S_{2z}\rangle_0|=2|\varepsilon_0|B/(\varepsilon_0^2+B^2)<1$. This is the effect of the magnetic anisotropy. As a result, the quantum fluctuations give a different value of the fluctuating moments along the axes x and y (principal axes for S_1), $\langle S_{1x}^2\rangle_0-\langle S_{1y}^2\rangle_0=(\varepsilon_0^2-B^2)/(\varepsilon_0^2+B^2)$. This leads to the dependence of interaction on a mutual orientation of the S_1 and S_2 anisotropy axes.

The interaction $H_{int}=-g\mathbf{S}_1\cdot\mathbf{S}_2$ gives nonzero matrix elements

$$V\equiv\langle 00|H_{int}|00\rangle=-\frac{4g\varepsilon_0^2B^2}{(\varepsilon_0^2+B^2)^2}, \tag{2}$$

$$P(\phi)\equiv\langle 00|H_{int}|11\rangle=\langle 11|H_{int}|00\rangle^* = g\left(\cos\phi-\frac{2i\varepsilon_0 B}{\varepsilon_0^2+B^2}\sin\phi\right). \tag{3}$$

Assuming $B\gg\lambda$, we can restrict ourselves by considering only the four lowest-energy states. Then the ground-state energy E_0 of the Hamiltonian (1) can be found by the diagonalization of a 4×4 matrix, and we obtain

$$E_0(\phi)=\tilde{\varepsilon}_0+\lambda/2-[(\tilde{\varepsilon}_0-\lambda/2)^2+|P(\phi)|^2]^{1/2}, \tag{4}$$

where $\tilde{\varepsilon}_0=\varepsilon_0+V/2$. Due to the exchange interaction, the energy depends on the mutual orientation of the anisotropy axis. The torque acting on the spins is $\mathcal{M}(\phi)=-dE_0/d\phi$. In the limit of $g\to 0$ we find from (4)

$$\mathcal{M}(\phi)\simeq-\frac{g^2(\varepsilon_0^2-B^2)^2\sin 2\phi}{2(\varepsilon_0^2+B^2)^2(\lambda/2-\varepsilon_0)}. \tag{5}$$

The torque vanishes for parallel or antiparallel configurations of the anisotropy axes, and also if they are perpendicular to each other, $\phi=\pi/2$.

Let us consider now a model with arbitrary spins $S\geqslant 1$ on a lattice, with a nearest-neighbor exchange interaction. We assume a one-site anisotropy with the anisotropy axis $\mathbf{n}_i$ smoothly depending on position. The Hamiltonian has the following form:

$$H=-\frac{g}{2}\sum_{<i,j>}\mathbf{S}_i\cdot\mathbf{S}_j+\frac{\lambda}{2}\sum_i(\mathbf{n}_i\cdot\mathbf{S}_i)^2. \tag{6}$$

We assume that in the ground state the ferromagnet has uniform magnetization along the axis z, and all vectors $\mathbf{n}_i$ lie in the x-y plane. This corresponds to an easy plane ferromagnet with a nonuniform anisotropy and can be realized, for example, in a thin-walled magnetic cylinder (Fig. 2).[8,10]

Using a local frame with the x axis along $\mathbf{n}_i$ in each lattice site, one transforms (1) into

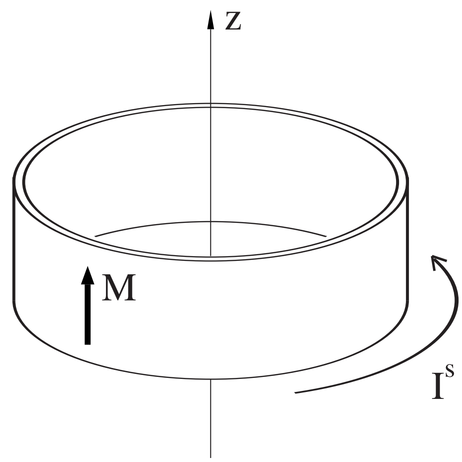

FIG. 2. Thin magnetic cylinder with homogeneous magnetization and noncollinear anisotropy.

$$H=-\frac{g}{2}\sum_{\langle i,j\rangle}S_i^Te^{i(\phi_i-\phi_j)J_z}S_j+\frac{\lambda}{2}\sum_i(S_i^x)^2, \tag{7}$$

where ϕ_i is the rotation angle, and we use a matrix form of presentation for the spin operators, $S_i^T=(S_i^x,S_i^y,S_i^z)$.

After the Fourier transformation $S_i^\mu=\Sigma_{\mathbf{q}}S_{\mathbf{q}}^\mu e^{i\mathbf{q}\cdot\mathbf{r}_i}$ and taking the limit of $q\to 0$, the Hamiltonian is

$$H=\frac{a}{2}\sum_{\mathbf{q}}[S_{-\mathbf{q}}^Tq^2S_{\mathbf{q}}+2i(\mathbf{q}\cdot\boldsymbol{\mathcal{A}})(S_{-\mathbf{q}}^xS_{\mathbf{q}}^y-S_{-\mathbf{q}}^yS_{\mathbf{q}}^x) + \boldsymbol{\mathcal{A}}^2(S_{-\mathbf{q}}^xS_{\mathbf{q}}^x+S_{-\mathbf{q}}^yS_{\mathbf{q}}^y)]+\frac{\lambda}{2}\sum_i(S_i^x)^2, \tag{8}$$

where $\boldsymbol{\mathcal{A}}=\boldsymbol{\nabla}_i\phi_i$ is the gauge potential, $a=ga_0^2$, a_0 is the lattice constant, and we assume that the variation of $\boldsymbol{\mathcal{A}}$ in space is small at the wavelength of magnons (adiabatic approximation).[8]

We use the Holstein-Primakoff representation of spin operators,[12] $S_i^z=S-n_i$, $S_i^+=\sqrt{2S}(1-n_i/2S)^{1/2}a_i$, $S_i^-=\sqrt{2S}a_i^\dagger(1-n_i/2S)^{1/2}$, $n_i=a_i^\dagger a_i$, and restrict ourselves by the harmonic approximation in a_i and $a_i^\dagger$. Here $a_i^\dagger$ and a_i are the boson creation and annihilation operators for the magnons. Then we find

$$H=\frac{S}{2}\sum_{\mathbf{q}}(\{[a(\mathbf{q}+\boldsymbol{\mathcal{A}})^2+\lambda_1](a_{\mathbf{q}}^\dagger a_{\mathbf{q}}+a_{\mathbf{q}}a_{\mathbf{q}}^\dagger)\} + \lambda_2(a_{\mathbf{q}}a_{-\mathbf{q}}+a_{\mathbf{q}}^\dagger a_{-\mathbf{q}}^\dagger)) \tag{9}$$

where $\lambda_1=\lambda(1-1/2S)$ and $\lambda_2=\lambda(1-1/2S)^{1/2}$.

In the following, we make a change in Eq. (9), substituting $\lambda_2\to\lambda_1=\lambda(1-1/2S)$. This is related to the well-known renormalization of the constant λ_2 due to the magnon interactions.[13] It reproduces correctly both the $S\to 1/2$ and $S\to\infty$ limits, and leads to the gapless spectrum of magnons in correspondence with the Goldstone theorem.[14] The Hamiltonian (9) with $\lambda_2\to\lambda_1$ can be diagonalized using the Bogoliubov-Holstein-Primakoff transformation method,[12] and we obtain finally

$$H=\sum_{\mathbf{q}}\omega_{\mathbf{q}}(b_{\mathbf{q}}^{\dagger}b_{\mathbf{q}}+1/2), \quad (10)$$

where the $b_{\mathbf{q}}^{\dagger}$ and $b_{\mathbf{q}}$ operators are related to $a_{\mathbf{q}}^{\dagger}$ and $a_{\mathbf{q}}$ by the Bogoliubov-Holstein-Primakoff transformation

$$b_{\mathbf{q}}=\alpha_{\mathbf{q}}a_{\mathbf{q}}+\beta_{\mathbf{q}}a_{-\mathbf{q}}^{\dagger}, \quad b_{\mathbf{q}}^{\dagger}=\alpha_{\mathbf{q}}^{*}a_{\mathbf{q}}^{\dagger}+\beta_{\mathbf{q}}^{*}a_{-\mathbf{q}}, \quad (11)$$

and $\omega_{\mathbf{q}}$ is the energy spectrum of magnons,

$$\omega_{\mathbf{q}}=S(\tilde{J}_{\mathbf{q}}-\tilde{J}_{-\mathbf{q}})+S[(\tilde{J}_{\mathbf{q}}+\tilde{J}_{-\mathbf{q}})(\tilde{J}_{\mathbf{q}}+\tilde{J}_{-\mathbf{q}}+2\lambda_1)]^{1/2}. \quad (12)$$

Here we denoted

$$\tilde{J}_{\mathbf{q}}\equiv J(0)-J(\mathbf{q})=a(\mathbf{q}+\boldsymbol{\mathcal{A}})^2 \quad (13)$$

and used the standard notation $J(\mathbf{q})$ for the Fourier transform of the exchange interaction.

Using (12) and (13) we can find the dependence of momentum $\mathbf{q}$ on $\boldsymbol{\mathcal{A}}$. It determines the Berry phase acquired by the spin wave, which moves in the gauge potential related to the varying anisotropy. We find that in the case of adiabatic motion (see details in Ref. 8), the Berry phase along a contour C is $\gamma_B(C)=\oint_C\tilde{\boldsymbol{\mathcal{A}}}(\mathbf{r})d\mathbf{r}$, where $\tilde{\boldsymbol{\mathcal{A}}}=\boldsymbol{\mathcal{A}}[1+S^2\lambda_1^2/(\omega_{\mathbf{q}}^0)^2]^{-1/2}$ is the effective gauge potential acting on magnons, and $\omega_{\mathbf{q}}^0=2S[aq^2(aq^2+\lambda_1)]^{1/2}$. For $\lambda_1\to 0$, we get $\tilde{\boldsymbol{\mathcal{A}}}=\boldsymbol{\mathcal{A}}$, but for a large anisotropy, $\lambda_1\gg\omega_{\mathbf{q}}^0/S$, the effective potential $\tilde{\boldsymbol{\mathcal{A}}}\simeq\boldsymbol{\mathcal{A}}\omega_{\mathbf{q}}^0/S\lambda_1$ is small compared to $\boldsymbol{\mathcal{A}}$. Thus, the Berry phase of magnons is determined by $\tilde{\boldsymbol{\mathcal{A}}}(\mathbf{r})$. This generalizes the result of Ref. 8 for arbitrary quantum spins on the lattice.

We consider now the contribution to the total energy of the vacuum fluctuations, $E_0=\Sigma_{\mathbf{q}}(\omega_{\mathbf{q}}/2)$, corresponding to the second term in (10). The energy E_0 depends on the gauge potential $\boldsymbol{\mathcal{A}}$ resulting from the inhomogeneous anisotropy. To calculate the spin current density $\mathbf{j}^s$ we add to $\boldsymbol{\mathcal{A}}$ an additional fictitious potential $\boldsymbol{\mathcal{A}}'(\mathbf{r})$, which we set to zero after calculation. Thus, in the previous definition of $\tilde{J}_{\mathbf{q}}$ we substitute $\boldsymbol{\mathcal{A}}\to\boldsymbol{\mathcal{A}}+\boldsymbol{\mathcal{A}}'$. Note that $\boldsymbol{\mathcal{A}}'$, like the original potential $\boldsymbol{\mathcal{A}}$, is associated with spin transformation—rotation around the axis z, which corresponds to the spin current density $\mathbf{j}^s$ with the spin polarization along z.

We use the definition $j_i^s(\mathbf{r})=\gamma[\delta E_0/\delta\mathcal{A}_i'(\mathbf{r})]_{\mathcal{A}'=0}$ following from the connection between the spin current conservation and the invariance with respect to rotations in the spin space,[8] where γ is the gyromagnetic ratio. Then we obtain

$$\mathbf{j}^s=\frac{2a\gamma S}{\Omega}\sum_{\mathbf{q}}\left(\mathbf{q}+\frac{\boldsymbol{\mathcal{A}}(\tilde{J}_{\mathbf{q}}+\tilde{J}_{-\mathbf{q}}+\lambda_1)}{[(\tilde{J}_{\mathbf{q}}+\tilde{J}_{-\mathbf{q}})(\tilde{J}_{\mathbf{q}}+\tilde{J}_{-\mathbf{q}}+2\lambda_1)]^{1/2}}\right), \quad (14)$$

where Ω is the volume. In the usual bulk or an infinite two-dimensional (2D) system, the first term vanishes due to the inversion symmetry $\mathbf{q}\to-\mathbf{q}$, and, taking the limit of $\boldsymbol{\mathcal{A}}\to 0$, we find

$$\mathbf{j}^s=\frac{a\gamma S\boldsymbol{\mathcal{A}}}{\Omega}\sum_{\mathbf{q}}\frac{2aq^2+\lambda_1}{[aq^2(aq^2+\lambda_1)]^{1/2}}. \quad (15)$$

Here the sum over momentum runs up to $q_{max}\simeq\pi/a_0$. We can estimate it for the 3D case as

$$\frac{1}{\Omega}\sum_{\mathbf{q}}\frac{2aq^2+\lambda_1}{[aq^2(aq^2+\lambda_1)]^{1/2}}\simeq\frac{1}{3\pi^2}[(q_{max}^2+\kappa^2)^{3/2}-\kappa^2] -\frac{\kappa^2}{2\pi^2}[(q_{max}^2+\kappa^2)^{1/2}-\kappa], \quad (16)$$

where $\kappa=(\lambda_1/a)^{1/2}$.

As follows from (15), the spin current flows along the gradient of the anisotropy axis variation. In terms of the continuous anisotropy field $\mathbf{n}(\mathbf{r})$, it can be written as $j_i^{s\mu}\sim\epsilon^{\mu\nu\lambda}n^{\nu}(\partial n^{\lambda}/\partial r_i)$, where the index μ refers to the spin polarization and $\epsilon^{\mu\nu\lambda}$ is the unit antisymmetric tensor. The spin current is responsible for the transmission of angular momentum and produces the torque rotating the anisotropy axis at each site to orient all of them in the same direction.

In the case of magnetic rings, the first term in (14) can be nonvanishing, which is related to the Berry phase acquired by a moment moving along the ring.[10] As we assume the magnetic ordering uniform, the Berry phase has a nongeometric origin but is determined by the topology of the ring.[10] In its turn, the nonvanishing Berry phase affects the inversion symmetry with respect to the motion along the ring in opposite directions.

To specify this suggestion, let us consider now the mesoscopic magnetic ring in the form of a thin-walled cylinder, with the magnetization along the z axis of cylinder, as described in Ref. 8. The hard axis is directed perpendicular to the cylinder surface. Correspondingly, it changes direction along a contour around the ring. We can restrict ourselves by considering only the φ component of momentum $\mathbf{q}$ corresponding to the around-ring motion (see Fig. 2) because the gauge potential in this case has only one nonvanishing component $\mathcal{A}_{\varphi}=1/R$, where R is the radius of the ring. Note that $\mathcal{A}_{\varphi}$ is a constant, and it fully justifies using the Fourier transformation leading to Eq. (8). The periodic condition imposes the requirement of momentum quantization, which reads as $(q_{\varphi n}+\tilde{\mathcal{A}}_{\varphi})\ R=n$, where $n=0,\pm1,\ldots$. Then the first term in Eq. (14) gives us the spin current in the ring

$$\mathcal{I}^s=\frac{a\gamma S}{\pi R^2}\sum_n\left[n-\left(1+\frac{S^2\lambda_1^2}{(\omega_n^0)^2}\right)^{-1/2}\right], \quad (17)$$

where $\omega_n^0=2S[(an^2/R^2)(an^2/R^2+\lambda_1)]^{1/2}$. In the limit of $\beta\equiv\lambda_1R^2/a\gg 1$, which corresponds to the weak gauge field regime, the main contribution is related to $n\sim\sqrt{\beta}\gg 1$, and we can substitute the sum in (17) by the integral. In this approximation, we obtain

$$\mathcal{I}^s\simeq\frac{C\gamma S(\lambda_1 a)^{1/2}}{\pi R}, \quad (18)$$

where

$$C = \int_{-\infty}^{\infty} dx \left(1 - \frac{2|x|\sqrt{x^2+1}}{\sqrt{1+4x^2(x^2+1)}} \right) \simeq 0.754. \tag{19}$$

As follows from (18), the spin current is zero if the anisotropy constant $\lambda_1=0$. By evaluating the second contribution of Eq. (14) to the spin current, we find that in this case it is small with respect to (18) in a parameter $a_0/(R\sqrt{\beta}) \ll 1$.

Using (18) we can estimate the magnitude of the mechanical stress in the ring. We take $\lambda_1=3.4\times 10^{-14}$ erg, corresponding to $\lambda_1/a_0^3=4\pi M^2$, $M=10^4$ G, $a_0=3\times 10^{-8}$ cm; $a/a_0^2=4\times 10^{-14}$ erg, $2R=10$ nm. Then we get the angular momentum $\mathcal{I}^a \simeq 3\times 10^{-16}$ erg. Varying the length of ring L by δL, we get a variation $|\delta\mathcal{I}^a|=(\mathcal{I}^a/L)\,\delta L$. Correspondingly, the force acting in the cross section of the ring is given by $f=\delta\mathcal{I}^a/\delta L=\mathcal{I}^a/L$, and the stress $\sigma=f/S_0$, where S_0 is the cross section of the ring. Taking $S_0=5\times 10^{-14}$ cm^2 (5×1 nm^2) we find $\sigma\simeq 0.05$ N/cm^2.

In conclusion, we demonstrated the presence of nonvanishing equilibrium spin currents in magnetic structures with homogeneous magnetization and varied anisotropy axis. One of the most important consequences of this effect is related to the correct definition of the nonequilibrium spin current and interpretation of spin current measurements.[15–18] The existing attempts to exclude the equilibrium spin current have concentrated only on the part responsible for the torque acting on the spins. In a collinear magnetic system, this part of the torque is zero. Nevertheless, as we demonstrated, the spin current can also transmit angular momentum, rotating the anisotropy axes as in the case of magnetic rings. Similar to the magnetoelectric effect in magnetically inhomogeneous systems,[9] the mechanical stress in anisotropy-inhomogeneous systems can be called the *magneto mechanical* effect.

Our estimation of the stress suggests that the corresponding deformations are too small to be measured experimentally. However, the spin current in the ring can be observed due to the electric polarization, as was discussed recently.[5,6,9,11] It should be noted that the combination of magnetoelectric and magnetoelastic effects in a ferromagnet can be connected with the physics of ferroic materials, which are a field scope of great activity now.[19]

We calculated the spin current at $T=0$, when the excitation of real magnons can be neglected. The effect of temperature is twofold. Due to the excitation of magnons, one can expect an additional contribution to the spin current, related to the Berry phase from the Dirac string.[10] On the other hand, the magnon decoherence increases with temperature due to the magnon-phonon interaction, and it suppresses the spin current.

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